

# THE PHYSICAL SOCIETY OF LONDON.

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## PROCEEDINGS.

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VOLUME 36.—PART 5.

AUGUST 15, 1924.

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1924



# THE PHYSICAL SOCIETY OF LONDON.

1924-25.

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# PROCEEDINGS

AT THE

## MEETINGS OF THE PHYSICAL SOCIETY OF LONDON

SESSION 1923-1924.

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Except where otherwise stated, the Meetings were held at the Imperial College of Science, under the Chairmanship of the President for the time being.

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October 26, 1923.

The following Papers were read :—

1. "The Fine Structure of Some Sodium Salts of the Fatty Acids in Soap Curds," by S. H. PIPER, D.S.O., B.Sc., and E. N. GRINDLEY, B.Sc.
  2. "X-ray Analysis of Solid Solutions," by E. A. OWEN, M.A., D.Sc., and G. D. PRESTON, B.A.
  3. "Cohesion," by Dr. H. CHATLEY.
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November 9, 1923.

A DEMONSTRATION of "Experiments on the Reproduction of Vowel Sounds," was given by Sir RICHARD PAGET.

The following Papers were read :—

1. "The Scattering of Light by Carbon Dioxide, Nitrous Oxide and Some Organic Vapours," by Prof. A. L. NARAYAN, M.A. (The Paper was presented by Lord Rayleigh.)
  2. "Measurement of the Surface Tension of a Small Quantity of Liquid," by Dr. ALLAN FERGUSON.
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November 29, 1923.

At the Institution of Electrical Engineers.

A joint DISCUSSION with the Institution of Electrical Engineers took place on the subject of "Loud-Speakers for Wireless and Other Purposes."

## PROGRAMME.

3-5 p.m. Visit to the Studio of the British Broadcasting Company.

5.30-7 p.m. FIRST SESSION OF DISCUSSION.

1. General Principles involved in the Accurate Reproduction of Sound by Means of a Loud-Speaker. By Prof. A. O. RANKINE.

2. Theory of Loud-Speaker Design. Some factors affecting faithful and efficient reproduction. Causes of inefficiency. Economy in amplifiers resultant on improved efficiency. With Demonstration. By Dr. L. C. POCKOCK.

3. The Electrical Side of the Phenomena of Low-Frequency Amplification in Wireless Circuits. Causes of distortion and means of elimination. Demonstration. By Prof. C. L. FORTESCUE.

4. The Problem of Distortion in the Reproduction of Sound by the Gramophone. By Mr. H. I. PORTER.

5. The general Discussion was opened by Dr. W. H. ECCLES, F.R.S.

7-8 p.m. Interval.

8-9.30 p.m. SECOND SESSION OF DISCUSSION.

6. Distortion in Audio-Circuits. Relative importance of various frequencies. Demonstration of effect of elimination of various frequency bands. Demonstration of nodes and anti-nodes in an auditorium. Methods, direct and indirect, of Measurement of the Efficiency and Faithfulness of Loud-Speaking Telephones. By Mr. E. K. SANDEMAN, B.Sc.

7. The Overtones of the Diaphragm of a Telephone Receiver. Their effect in causing distortion. With Demonstration. By Prof. J. T. MCGREGOR-MORRIS and Prof. E. MALLET.

8. Architectural Acoustics. Its relation to Loud-Speaker operation. By Mr. G. A. SUTHERLAND, M.A.

9. Some Directions of Improvement of the Loud-Speaking Telephone. Action of the Frenophone. With Demonstration. By Mr. S. G. BROWN, F.R.S.

10. The Characteristics of the Gaumont-Lumière Loud-Speaker. With Demonstration. By Capt. P. P. ECKERSLEY.

11. The general Discussion was adjourned till February 14.

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December 14, 1923.

The following Papers were read :—

1. "The Aerodynamic Resistance of Spheres Shot Upward to Measure the Wind," by L. F. RICHARDSON, F.Inst.P., F.R.Met.Soc.



2. "X-Ray Analysis of Zinc-Copper Alloys," by E. A. OWEN, M.A., D.Sc., and G. D. PRESTON, B.A.

3. "Investigations of Piezo-Electric Effects with Dielectrics," by K. R. BRAIN, B.Sc.

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January 2 and 3, 1924.

The Annual Exhibition of Apparatus was held by the Physical Society of London and the Optical Society from 3-6 p.m. and 7-10 p.m. each day.

Discourses were given as follows :—

On January 2nd at 4 p.m. and January 3rd at 8 p.m. : "The Heaps and Grylls Rapid Cinema Machine" (designed to take photographs at rates varying from 500 to 5,000 times per second), by Mr. H. B. GRVLLS.

On January 2nd at 8 p.m. and January 3rd at 4 p.m. : "The Nature and Artificial Production of Human Speech (Vowel Sounds)," by Sir RICHARD PAGET, Bart.

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January 25, 1924.

A Lecture entitled "Recent Work in Stellar Physics" was delivered by E. A. MILNE, M.A., Fellow of Trinity College, Cambridge, and Assistant-Director of the Solar Physics Observatory, Cambridge.

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February 8, 1924.

*Annual General Meeting.*

#### GENERAL BUSINESS.

The Report of the Council and that of the Treasurer were presented and unanimously adopted.

#### REPORT OF THE COUNCIL.

Twelve ordinary Science Meetings have been held at the Imperial College of Science during the year. At these meetings 21 Papers were presented to the Society and 7 Demonstrations were given.

In addition the Society co-operated in two Joint Discussions—one with the Röntgen Society and the other with the Institution of Electrical Engineers. The former was on the subject of "X-ray Measurements," and was held at the Imperial College on February



23, there being present about 120 Fellows and Visitors. It was opened by Prof. Sir William Bragg, F.R.S.; 4 Papers were read and 6 Demonstrations were given.

The Joint Discussion with the Institution of Electrical Engineers was on "Loud-Speakers for Wireless and Other Purposes." It was held on November 29 in the large theatre of the Institution, which was crowded throughout the proceedings. The opener was Prof. A. O. Rankine. Altogether 10 Papers were read, some of them being illustrated by interesting experiments. In view of the interest aroused by this discussion an agreement has been reached with the Institution of Electrical Engineers to continue it at the Institution on February 14, 1924.

On May 11 Mr. J. H. Jeans, Sec. R.S., delivered the Eighth Guthrie Lecture on "The Present Position of the Radiation Problem." The lecture was attended by 156 Fellows and Visitors.

Two other lectures formed part of the Proceedings of the Society during the year. On June 8 Prof. James G. Gray gave a Lecture on "A General Solution of the Problem of Finding the True Vertical for all Types of Marine and Aerial Craft," illustrated by experiments; and on June 22 Prof. F. Horton, F.R.S., delivered a Lecture on "The Excitation and Ionisation Potentials of Gases and Vapours."

Apart from the Joint Discussions and the Guthrie Lecture the average attendance at the meetings of the Society was 62.

The Thirteenth Annual Exhibition of Scientific Apparatus, arranged jointly by the Physical and Optical Societies, was held, through the courtesy of the Governing Body, at the Imperial College on January 3 and 4. Fifty-six firms took part in the Exhibition, which was one of the most successful ever held, the total number of visitors being about 3,000. The Discourses, which were given twice, proved a great attraction. Mr. W. Gamble lectured on "Reproduction of Colour by Photographic Processes," and Prof. E. G. Coker, F.R.S., on "Recent Photo-Elastic Researches on Engineering Problems."

At the celebrations of the Jubilee of the Société Française de Physique in November, the Treasurer, Mr. W. R. Cooper, represented the Society as official delegate, and presented on behalf of the Society an Address of Congratulation in the following terms:—

#### TO THE SOCIÉTÉ FRANÇAISE DE PHYSIQUE.

On the Occasion of the 50th Anniversary of Its Foundation.

Looking back on those years which saw the dawn of modern science, the illustrious names of Descartes, Laplace, Fourier, Poisson, Fresnel, Ampère bear witness to the invaluable work done by France in laying the foundations of Physics. Progress in more recent times has been largely due to the activity of the Société Française de Physique which gathered within its early membership Amagat, the Becquerels, Cornu, Fizeau, Mascart, Regnault, and many others whose names are household words throughout the world.

Their work has largely extended the boundaries of our knowledge. Their brilliant discoveries have been rivalled by those of the present generation of members, many of whom have attained world-wide fame.

That the work of the Société Française de Physique may flourish in the future as it



has during the past 50 years is the sincere wish of the Physical Society of London, and may it ever form a bond of union between the two sister nations.

On behalf of The Physical Society of London.

ALEXANDER RUSSELL, *President.*

W. R. COOPER, *Treasurer.*

ARTHUR SCHUSTER, *Foreign Secretary.*

DAVID OWEN, }  
A. O. RANKINE, } *Secretaries.*

Mr. T. Smith and Dr. D. Owen have been appointed to represent the Society on the National Committee for Physics ; Mr. F. E. Smith and Prof. C. I. Fortescue on the National Committee for Radio-telegraphy ; Mr. C. E. S. Phillips and Mr. F. E. Smith on the Board of the Institute of Physics ; and Mr. F. J. W. Whipple on the Geophysical Committee of the Royal Astronomical Society.

The Council has agreed to co-operate with the Röntgen Society in the formation of a Joint Committee to inquire into the possibility of establishing an X-ray unit of quantity for scientific purposes. Prof. Sir William Bragg, Dr. E. A. Owen and Dr. F. L. Hopwood have been appointed to represent the Physical Society on this Committee.

The Council is glad to be able to report that satisfactory arrangements have now been made with the Institution of Electrical Engineers and with the American Physical Society with regard to the printing of Section A of "Science Abstracts." The improvement in the financial position of "Science Abstracts" has made it possible to reduce the annual price of Section B to Fellows to its former amount—five shillings. The Society's Treasurer, Mr. W. R. Cooper, is the Editor, and the Science Abstracts Committee is doing its utmost to increase the usefulness of this publication.

The new edition of Mr. Jeans' "Report on the Quantum-Theory" has been issued and has been very favourably received.

Previous to the delivery of the Guthrie Lecture on May 11, Sir William Bragg, on behalf of the Duddell Memorial Committee, handed over the Memorial Medal and funds to the President. The Council has decided unanimously to make the first award of this medal to Prof. H. L. Callendar, F.R.S., and to present it to him at the annual general meeting.

In October, owing to pressure of work, Mr. F. E. Smith, C.B.E., F.R.S., was compelled to resign his post as Business Secretary. The Council accepted his resignation with great regret and gave him their warmest thanks for the valuable work he had done during the last five years for the Society. Professor A. O. Rankine was appointed his successor.

The Council has given consideration to the mode of celebration of the Jubilee of the Society, which falls on March 21, 1924. Preliminary steps have been taken and a sub-committee has been appointed to make the requisite arrangements.

The Society has to record with regret the deaths of Mrs. Hertha Ayrton, M.I.E.E., Sir Lazarus Fletcher, F.R.S., Dr. J. A. Harker, F.R.S., Mr. S. A. Notcutt, Mr. J. W. Russell, Mr. J. E. Stead, F.R.S., Sir W. H. Christie, F.R.S. and Prof. J. A. Pollock, F.R.S. Mrs. Ayrton was elected a Fellow in 1907. Sir Lazarus Fletcher was a Life Fellow elected in 1882, and had served as a Vice-President. Dr. Harker became a Fellow in 1897, and served on the Council from 1906-1910. Both Mr. Notcutt and Mr. Russell were Life Fellows, the former having been elected in 1895 and the latter in 1878, four



years after the Society's formation. Mr. Stead also was a Life Fellow, and was elected in 1881. Sir W. H. Christie was one of the earliest Life Fellows, having been elected in 1877, and he had served on the Council. Prof. Pollock became a Life Fellow in 1899.

The number of Honorary Fellows on the Roll on December 31, 1923, was ten, Dr. S. W. Stratton, President of the Massachusetts Institute of Technology, having been elected an Honorary Fellow at the last Annual General Meeting. The number of ordinary Fellows and Students was 597, which constitutes a record. During the year, thirty-two new Fellows and five Students were elected, and there were two resignations.

#### REPORT OF THE TREASURER.

The accounts this year again show a normal position, the balance of income over expenditure being £82 9s. 10d., as compared with a debit balance of £60 13s. 9d. in the previous year. This balance would have been larger but for certain unusual items of expenditure, more particularly the contribution of £50 to the Duddell Memorial Committee. The amount spent on the ordinary publications has increased, the increase in the case of the "Proceedings" being £125 18s. 1d.; the expenditure on special publications has been much less. The total expenditure has decreased by £49 1s., and part of the expenditure is simply a book transfer of dividends.

Turning to the revenue side, the amount realised by sales has fallen by £39 17s. The whole position has been somewhat affected by the publication of Mr. Jeans' Report on Radiation quite late in the year, so that the cost thereof comes into these accounts without any corresponding sales. The total income has increased by £94 3s. 1d.

The assets of the Society have been increased by the Duddell Memorial Fund, which was handed over by the Committee during the year. In addition to being included in the balance-sheet, this is set out separately, together with the income therefrom, so that the state of the fund is clearly apparent. There is no separate banking account for this fund, and therefore the income is shown in the balance-sheet with the cash at bank. The investments have been valued at market prices through the courtesy of the Manager of the Charing Cross Branch of the Westminster Bank. Certain alterations appear owing to the railway amalgamations, but the total value of the stocks brought forward happens to remain at precisely the same figure as in the last accounts. During the year the sum of £100 16s. 3d. was invested in the purchase of £100 5 per cent. War Loan.

#### ELECTION OF OFFICERS AND COUNCIL.

The following Officers and Members of Council were elected for 1924 :—

*President* : F. E. Smith, C.B.E., F.R.S.

*Vice-Presidents* : E. H. Rayner, M.A., Sc.D.; J. H. Vincent, D.Sc., M.A.; T. Smith, B.A.; C. R. Darling, F.I.C.

*Hon. Secretaries* : D. Owen, B.A., D.Sc.; Prof. A. O. Rankine, O.B.E., D.Sc.

*Hon. Foreign Secretary* : Sir Arthur Schuster, Ph.D., Sc.D., F.R.S.

*Hon. Treasurer* : W. R. Cooper, M.A., B.Sc.

*Hon. Librarian* : J. H. Brinkworth, M.Sc., A.R.C.Sc.

*Other Members of Council* : R. W. Paul; Prof. C. L. Fortescue, O.B.E.; W. S. Tucker, D.Sc.; Prof. S. W. J. Smith, M.A., D.Sc., F.R.S.; J. S. G. Thomas, D.Sc.; J. Guild, A.R.C.Sc., D.I.C.; F. L. Hopwood, D.Sc.; E. A. Owen, B.A., D.Sc.; J. Robinson, M.Sc., Ph.D.; G. B. Bryan, D.Sc.







# BALANCE SHEET AT DECEMBER 31ST, 1923.

ASSETS.		LIABILITIES.	
	£ s. d.		£ s. d.
Subscriptions in arrears .....	72 10 3	Life Compositions .....	2,038 10 0
Less reserve for subscriptions probably unrealisable .....	31 10 0	W. F. Stanley Trust Fund .....	420 0 0
		Duddell Memorial Trust Fund .....	426 12 9
Investments (valued at Dec. 31):—			
£399 London Midland & Scottish Railway 4 per cent. Debenture Stock .....	320 0 0		
£1,000 London Midland and Scottish Railway 4 per cent. Preference Stock .....	780 0 0		
£200 Metropolitan Board of Works 3½ per cent. Consolidated Stock .....	186 0 0		
£400 Lancaster Corporation 3 per cent. Redeemable Stock .....	284 0 0		
£254 2s. 9d. New South Wales 3½ per cent. Ordinary Stock .....	254 0 0		
£300 Southern Railway Preferred Ordinary Stock .....	252 0 0		
£442 Southern Railway Deferred Ordinary Stock .....	168 0 0		
£500 London & Eastern Railway 4 per cent. Debenture Stock .....	400 0 0		
£500 India 3½ per cent. Stock .....	325 0 0		
£850 4% Funding Loan, 1960-90 .....	559 0 0		
£300 5% War Loan 1929/47 inscribed, "A" Account (£100 purchased for £100 16s. 3d. in 1923) .....	300 0 0		
£400 5% War Loan 1929/47 inscribed, "B" Account (Duddell Memorial Fund) .....	400 0 0		
Stock of Publications (Treasurer's valuation) .....	4,228 0 0		
Cash at Bank, on Deposit .....	400 0 0		
Cash at Bank, Current Account .....	175 9 1		
Adjustment for outstanding cheques, &c. ....	325 9 1		
	197 3 8		
Duddell Memorial Fund, Revenue Balance .....	128 5 5		
Cash in hand (Treasurer's Petty Cash) .....	26 12 9		
	154 18 2	Balance, General Fund .....	1,941 5 4
	1 9 8		£4,826 8 1
	£4,826 8 1		

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct,  
R. L. SMITH-ROSE  
F. J. W. WHIPPLE } *Honorary Auditors.*

January 31st, 1924.



128 Fellows paid £10 .....	£	s.	d.
3 Fellows paid £15 .....	1,280	0	0
1 Fellow paid £20 10s .....	45	0	0
12 Fellows paid £21 .....	20	10	0
14 Fellows paid £31 10s .....	252	0	0
	441	0	0
158	£2,038	10	0

W. F. STANLEY TRUST FUND (FOR THE "BULLETIN").

£300 Southern Railway Preferred Ordinary Stock	£	s.	d.
£442 Southern Railway Deferred Ordinary Stock	252	0	0
	168	0	0
	£420	0	0
Carried to Balance Sheet.....	420	0	0
	£420	0	0

DUDELL MEMORIAL TRUST FUND.

CAPITAL.			
£400 War Loan 5% 1929/47 inscribed Stock.....	£	s.	d.
	400	0	0
	Carried to Balance Sheet .....		
	400	0	0
REVENUE.			
Cash Received from Duddell Memorial Committee	£	s.	d.
Dividends .....	6	12	9
	20	0	0
	£26	12	9

Audited and found correct,

W. R. COOPER, *Honorary Treasurer.*      R. L. SMITH-ROSE } *Honorary Auditors.*  
January 31st, 1924.      F. J. W. WHIPPLE }



## ELECTION OF AN HONORARY FELLOW.

PROFESSOR MAX PLANCK was elected an Honorary Fellow of the Society.

## AWARD OF THE DUDDELL MEMORIAL MEDAL.

On behalf of the Council, Dr. Alexander Russell awarded the first Duddell Memorial Medal to PROFESSOR H. L. CALLENDAR, C.B.E., M.A., LL.D., F.R.S., and made the following remarks :—

It is now my pleasing duty to make the first award of the Duddell Medal. You will remember that this medal is awarded by the Council to persons who have contributed to the advancement of knowledge by the invention or design of scientific instruments or by the discovery of the materials used in their construction. The award is open to the whole world. Nevertheless, the Council had little difficulty in making the award. They unanimously agreed to award the first Duddell Medal to Professor H. L. Callendar, C.B.E., M.A., LL.D., F.R.S.

I remember Professor Callendar quite well in my student days at Cambridge. We were in the same year and we went to Routh's together. I was tremendously impressed by the fact that, although science was his main subject, he took a first class in Classics and was a Wrangler apparently with the greatest ease.

I must confine myself, however, to the work he has done within the terms governing the award of this medal.

I. (a) The Electrical Resistance Thermometer, first communications to the Royal Society, 1886 and 1887. Prof. Callendar's construction and his investigations resulted in the present form of instrument. They have given to the research physicist the most perfect means of determining temperatures with great accuracy. At the same time, they have given to industry a simple, trustworthy and widely used method of temperature measurement.

In connection with the resistance thermometer, though capable of many other uses, he developed the convenient Callendar-Griffiths resistance bridge and various forms of continuous recorders.

(b) The Electrical Continuous Flow Calorimeter, also devised in 1886, though the first published account was given to the British Association at Toronto in 1897 and a complete account was only given in 1902 to the Royal Society. The elimination of the water equivalent and the simple method of determining the radiation remove the two great difficulties in calorimetric determinations, and constitute an advance second only to that of the perfection of the electrical resistance thermometer. In addition to Callendar's own work, the method has rendered possible that of many others, such as the classical work on the specific heat of water completed by his assistant, Prof. Barnes. Reversing the method, Callendar devised the compensated electric air flowmeter, described in recent reports of the Aeronautical Research Committee, the only direct reading instrument so far devised which will measure a flow of gas of steady or varying character whose indications are independent of pressure and temperature.

(c) Among other instruments of more limited application due to him and showing his inimitable genius for devising means of accurate measurement are the compensated air thermometer described to the Royal Society in 1891, and the Radio Balance described before the Physical Society in 1910.



II. Other research work which would specially have appealed to Duddell, i.e., research resulting in benefits to industry :—

(a) His formulation of the Callendar steam equation. This was first enunciated to the Royal Society in 1900. His steam tables deduced from it have proved of the greatest value to industry. They are accepted as standard by all steam users, both in this country and abroad. His researches on steam at high pressures and temperatures are still being carried forward at the direct request of steam turbine manufacturers and the Electrical Research Association.

(b) His experiments on the indication of petrol engines were the immediate cause of the work which resulted in the Watson optical indicator for high-speed internal-combustion engines, and he has contributed important Papers to the engineering societies on the power of internal-combustion engines.

III. In addition, he has contributed many other Papers of great importance to the Royal and Physical Societies, the *Encyclopædia Britannica*, the *Philosophical Magazine*, &c., on thermometric scales, radiation, vapour pressure, and practically every branch of the subject of heat.

I now have the pleasure of giving to Prof. Callendar the Duddell Medal, a cheque, and the certificate.

DEATH OF AN HONORARY FELLOW.

Dr. RUSSELL announced with great regret the death of an Honorary Fellow, Prof. G. H. Quincke. It was resolved that a message of condolence should be sent to his family.

VOTES OF THANKS.

The following votes of thanks were carried by acclamation : To the Hon. Auditors, Messrs. R. L. Smith-Rose and F. J. W. Whipple (proposed by Dr. J. H. Vincent and seconded by Mr. T. Smith) ; to the retiring Officers and Council (proposed by Dr. J. S. G. Thomas and seconded by Mr. R. S. Whipple) ; and to the Governors of the Imperial College of Science (proposed by Dr. E. H. Rayner and seconded by Mr. R. W. Paul).

ORDINARY MEETING FOLLOWING THE ANNUAL GENERAL MEETING.

The following Papers were read :—

1. "Æolian Tones," by E. G. RICHARDSON, M.Sc., East London College.
2. "Effect of Torsion on the Thermal and Electrical Conductivities of Metals," by J. E. CALTHROP, B.A., B.Sc., East London College.

A DEMONSTRATION of a "Microscope for Observation of Interference Fringes" was given by C. W. HAWKSLEY.

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February 14, 1924.

At the Institution of Electrical Engineers.

An adjourned Discussion on Loud-Speakers for Wireless and other Purposes was held jointly with the Institution of Electrical Engineers.

C. C. PATERSON, O.B.E., M.Inst.C.E., M.Inst.E.E., in the Chair.

The discussion was continued by Sir Richard Paget, Bart., Capt. B. S. Cohen, Mr.



G. H. Nash, Capt. H. J. Round, Messrs. W. J. Brown, A. H. Davies, A. J. Aldridge, G. C. Marris, W. E. Burnand, Capt. N. Lea, Dr. H. M. Barlow, and Mr. P. G. A. H. Voigt, Capt. Cohen and Mr. Voigt illustrated their remarks with demonstrations.

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February 22, 1924.

The following Papers were read :—

1. "A Generalisation of Whitehead's Theory of Relativity," by G. TEMPLE, B.Sc., Birkbeck College.
2. "The Structure of the Atomic Nucleus and the Mechanism of its Disintegration," by Dr. HANS PETTERSSON, Copenhagen.
3. "The Effect of a Magnetic Field on the Surface Tension of a Liquid of High Permeability," by Miss WINIFRED ROLTON, B.Sc., and Mr. R. STANLEY TROOP, B.Sc., East London College.

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March 14, 1924.

The following Papers were read :—

1. "A New Photo-Electric and Ionisation Effect," by U. A. OSCHWALD, B.Sc., and A. G. TARRANT, B.Sc.
2. "On Certain Properties of the Osglim Neon-Filled Lamp," by J. H. SHAXBY B.Sc., and E. J. EVANS, B.Sc.
3. "Notes on Some Electrical Properties of the Neon Lamp," by U. A. OSCHWALD, B.Sc., and A. G. TARRANT, B.Sc.
4. "The Critical Resistance for Flashing of the Low-Voltage Neon Discharge Tube," by J. TAYLOR, B.Sc., and W. CLARKSON, B.Sc.

DEMONSTRATIONS were shown in connection with the Papers.

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#### MEETINGS IN CELEBRATION OF THE SOCIETY'S JUBILEE.

On Thursday, March 20, 1924, at the Institution of Electrical Engineers (by kind permission of the Council of the Institution), the proceedings were opened at 3 p.m. with an address by the President, F. E. Smith, F.R.S. This was followed by the reception of delegates from kindred Foreign, Dominion and Home Societies, and the presentation of addresses. Thirty-one Societies in all were represented. Numerous telegrams of congratulation were received from foreign physicists.

M. le Duc de Broglie delivered the NINTH GUTHRIE LECTURE on "The Photo-electric Effect in the Case of High-frequency Radiation, and Some Associated Phe-

nomena." A vote of thanks to the Lecturer was proposed by Lord Rayleigh and seconded by Sir Ernest Rutherford.

At 6 p.m., under the auspices of the Institution of Electrical Engineers, an address on "The Nature of Speech" was given by Sir Richard Paget, at which many Fellows of the Society were present.

On Friday, March 21, at the Institute of Electrical Engineers, meetings were held from 4 to 5.30 p.m., and from 6 to 8 p.m. The speakers were as follows: Sir William Barrett, Prof. J. A. Fleming, Sir Richard Glazebrook, Sir Arthur Schuster, Prof. H. E. Armstrong, Dr. C. Chree and Sir Oliver Lodge. In the enforced absence of Prof. C. V. Boys a communication from him was read by the Secretary. Votes of thanks to the speakers were moved by Prof. W. Wien and Prof. S. J. M. Allen, and carried by acclamation.

An Exhibition of Apparatus was held at the Institute of Electrical Engineers during Thursday, Friday and Saturday, March 20 to 22. Demonstrations were given by Prof. C. R. Darling, Dr. F. L. Hopwood, the Western Electric Co., the Post Office Research Laboratory Staff, Major C. E. S. Phillips, Messrs. Hilger, Mr. J. T. Irwin and Mr. W. J. Colebrook.

On Saturday, March 22, a Banquet was held at the Connaught Rooms. The speakers included H.R.H. the Duke of York, Mr. F. E. Smith (President), the Right Hon. J. Ramsay MacDonald, Lord Haldane, Prof. Fabry, Sir J. J. Thomson, Sir Richard Glazebrook, Sir Ernest Rutherford, Sir Joseph Cook and Mr. J. H. Jeans.

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April 11, 1924.

The following Papers were read:—

1. "On the Thermo-electric Properties of Bismuth Alloys, with Special Reference to the Effect of Fusion. With a Note on Thermo-electric Inversion," by C. R. DARLING, F.I.C., F.Inst.P., and R. H. RINALDI, Finsbury Technical College. (The Paper was accompanied by a DEMONSTRATION.)

2. "Preliminary Measurement of a Primary Gas-grown Skin," by J. J. MANLEY, M.A., Magdalen College, Oxford.

3. "Removal of Gas-grown Skins from a Sprengel Pump," by J. J. MANLEY, M.A.

A DEMONSTRATION of Sub-harmonics produced by a Tuning fork was given by W. N. BOND, M.Sc., University College, Reading.

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May 9, 1924.

The following Papers were read:—

1. "The Diametral Plane in Elementary Optics," by CHARLES H. LEES, D.Sc., F.R.S., East London College.



2. "Apparatus for the Determination of the Latent Heats of Liquids of High Boiling Points," by J. H. AWBERY, B.A., B.Sc., and EZER GRIFFITHS, D.Sc., National Physical Laboratory.

3. "On the Study of Diffusion in Liquids by an Optical Method," by B. W. CLACK, Ph.D., Birkbeck College, London.

A DEMONSTRATION of a "Method of Optical Projection of Opaque Objects" was given by Mr. W. A. BENTON.

May 23, 1924.

The following Papers were read :—

1. "The Magnetism of Annealed Carbon Steels," by Prof. S. W. J. SMITH, F.R.S., A. A. DEE, B.Sc., and W. V. MAYNEORD, M.Sc., University of Birmingham.

2. "Some Thermo-Magnetic Properties of Nearly Pure Iron." Part 1 by W. V. MAYNEORD, M.Sc.; Part 2 by A. A. DEE, B.Sc.

3. "The Atomic Structure of Two Intermetallic Compounds," by E. A. OWEN, D.Sc., and G. D. PRESTON, B.A., National Physical Laboratory.

A DEMONSTRATION of "An Anomaly in Frictional Electricity" was given by Prof. A. O. RANKINE, Imperial College of Science and Technology.

June 13, 1924.

The following Papers were read :—

1. "On a Method for the Synchronous and Instantaneous Illumination of Objects Rotating or Vibrating at Very High Speeds," by Dr. G. E. BAIRSTO, Royal Aircraft Establishment, Farnborough.

2. "The Absorption and Scattering of  $\gamma$ -Rays," by Dr. E. A. OWEN, Mr. N. FLEMING and Miss W. E. FAGE, The National Physical Laboratory.

3. "The Flow of Compressible Fluids, Treated Dimensionally," by Mr. W. N. BOND, M.Sc., University College, Reading.

4. "Note on Israj, a Remarkable Indian Stringed Instrument," by Mr. D. B. DEODHAR, University of Lucknow, India.

*Proceedings of the Physical Society.*

xxi.

June 27, 1924.

Meeting held at King's College, Strand, W.C.2.

The following Papers and Demonstrations were presented :—

1. "Thermionic Emission from Systems with Multiple Thresholds," by Prof. O. W. RICHARDSON, F.R.S., King's College, University of London.

A DEMONSTRATION of "A Selenium Photometer," by Dr. F. C. TOY.

2. "A Method for Measuring very Small Capacities," by L. HARTSHORN, D.I.C., National Physical Laboratory.

A DEMONSTRATION of "A New String Galvanometer," by Prof. ERNEST WILSON, King's College.

A DEMONSTRATION of "The Fluorescence of Transparent Fused Silica," by Dr. W. E. CURTIS, King's College.

On the motion of the President a vote of thanks was accorded to Prof. O. W. Richardson and the Physics Staff of King's College for their hospitality and their contributions to the proceedings.

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July 19, 1924.

A SPECIAL MEETING of the Society was held at Cambridge, by kind invitation of Sir ERNEST RUTHERFORD and of the Directors of the CAMBRIDGE INSTRUMENT COMPANY.

PROGRAMME.

10.30 a.m. Visit to the Works of the Cambridge and Paul Instrument Company.

1.0 p.m. Lunch at Trinity College.

The President expressed the thanks of the Society to the Directors of the Cambridge Instrument Company and the Authorities of Trinity College. Sir HORACE DARWIN, K.B.E., F.R.S., and Sir J. J. THOMSON, O.M., F.R.S., replied.

2.45–3.45 p.m. Inspection of the Cavendish Laboratory, with Demonstrations of Experiments in progress as follows: (1) Positive Ray and other apparatus (Sir J. J. THOMSON). (2) Apparatus used in experiments on Atomic Disintegration (Prof. Sir E. RUTHERFORD and Dr. CHADWICK). (3) Mass Spectograph (Dr. ASTON). (4) Apparatus for the Production of Large Magnetic Fields (Dr. KAPITZA). (5) Apparatus of historic interest (Dr. WOOD). (6) Teaching and class apparatus (Dr. SEARLE). (7) Wilson Cloud Apparatus, showing  $\alpha$ -particle tracks (Mr. BLACKETT and Mr. GURNEY). (8) Photographs of atomic tracks (Mr. C. T. R. WILSON). (9) Apparatus used in



determination of  $\gamma$ -ray wave-lengths (Mr. C. D. ELLIS). (10)  $\alpha$ -particle counter (Dr. APPLETON and Mr. EMELEUS).

4.0 p.m. Tea at the Cavendish Laboratory.

4.15–5.15 p.m. Meeting at the Cavendish Laboratory. The following communications were given :—

1. "Radiations in a Discharge Tube," by Sir J. J. THOMSON, O.M., F.R.S.
2. "Recent Experiments on the Artificial Disintegration of the Elements," by Sir ERNEST RUTHERFORD, F.R.S., and Dr. J. CHADWICK.
3. "A Two-Dimensional Recording Accelerometer for Aeroplane Research," by Dr. G. F. C. SEARLE, F.R.S.

The President expressed the thanks of the Society for their reception in the Cavendish Laboratory, and Sir Ernest Rutherford, F.R.S., replied.

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# XXIII.—THE ATOMIC STRUCTURE OF TWO INTERMETALLIC COMPOUNDS.

By E. A. OWEN, M.A., D.Sc., and G. D. PRESTON, B.A., The National Physical Laboratory.

Received February 13, 1924.

## ABSTRACT.

The two intermetallic compounds  $Mg_2Si$  and  $AlSb$  have been examined by the X-ray spectrometer with the following results:—

$Mg_2Si$ .—A face centred cubic lattice of silicon atoms of side  $6.391\text{\AA}$  symmetrically intermeshed with a simple cubic lattice of magnesium atoms of side  $3.19\text{\AA}$ . There are eight magnesium atoms situated within each face centred cube of silicon atoms, dividing the four cubic diagonals in the ratio 1 : 3 and 3 : 1. Density from X-ray data  $1.95 \pm 0.05$  gms./c.c. The magnesium atoms are separated by the same distance as in the pure metal. The sum of the "radii" of silicon and magnesium atoms is equal to the distance between these atoms in the compound.

$AlSb$ .—A face centred cubic lattice of antimony of side  $6.126\text{\AA}$  intermeshed with an identical lattice of aluminium atoms; the corner of the latter dividing the cubic diagonal of the former in the ratio of 1 : 3. Density from X-ray data  $4.23 \pm 0.04$  gms./c.c. The molecular volume of the compound is greater than the sum of the atomic volumes of its constituents. The increase in volume observed by the method of X-ray analysis agrees with that obtained by other methods of measurement. The volume of the lattice on which the antimony atoms are arranged is 4 per cent. less than the volume of the face centred rhomb found in pure antimony. The molecular volume of the compound is accordingly 4 per cent. less than twice the atomic volume of antimony. The closest distance of approach of aluminium and antimony atoms is  $2.65\text{\AA}$ , which is less than the sum of their "radii" as determined from the pure metals.

VERY few intermetallic compounds have hitherto been examined by X-ray methods. Generally their study is somewhat difficult on account of the small intensities of the reflected beams and the difficulty of measuring the intensities of the reflections. The compounds, the structures of which were examined in the present investigation, presented no particular difficulty in this respect. The procedure, which we need only outline briefly here as it has already been described in detail in a previous Paper,\* is to mount on the spectrometer table a plate of the material containing crystals as small as can be produced. The plate is fixed in the position in which it gives the maximum amount of reflection for one definite line in the spectrum. In this position there will be a number of the small crystals at the proper orientation to reflect the radiation of the wave-length employed. Also, if each position of the plate is equally possible on account of the random distribution of the crystals, the particular position chosen will be the one in which there is the least amount of absorption and the best focusing of the radiation.

The plate is then rotated at half the rate of rotation of the ionisation chamber. The spectral curves obtained in this way show a number of definite maxima, the bases of which are generally situated on a smooth curve representing the distribution of intensity in the general radiation. The intensities of the maxima have been

\* Owen and Preston, Proc. Phys. Soc., Vol. 35, Part 2, p. 101, Feb. (1923).



determined by subtracting the ordinate of the smooth curve from that of the peak in each case. Although the intensity of the peak thus obtained is not always accurate as it is sometimes difficult to decide the course of the smooth curve with certainty, yet it provides in most cases sufficient information to guide one in the allocation of definite positions of the atoms in the lattice.

To determine the crystal structure of any material, we have first of all to determine the type of lattice present. This is done in the usual way by plotting

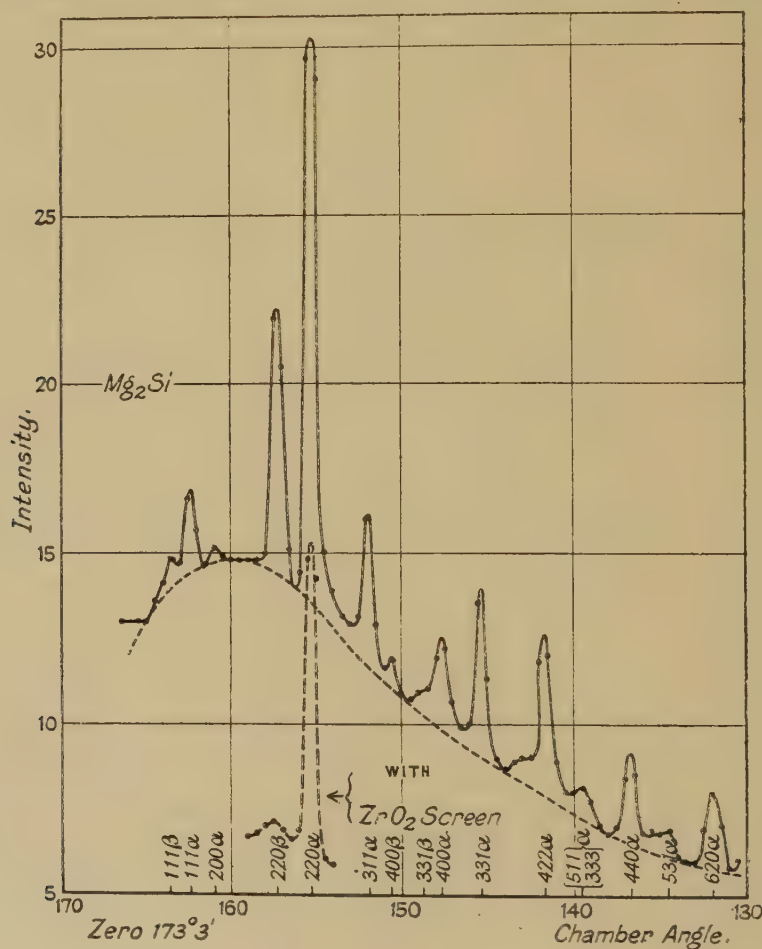


FIG. 1.

the lines observed in the spectrum and fitting them on suitably prepared graphs. The number of atoms associated with each unit of the lattice has to be such that the density calculated from the X-ray data agrees with the observed density of the material. Finally, it is necessary to measure the intensities of the peaks in the manner outlined above and thereby arrive at the relative intensities of the different order spectra. This enables us to fix the atoms in their respective positions in

the lattice. For the first of these determinations it is essential that the plate should consist of a large number of small crystals, so that within the region examined each crystal plane shall be represented. Since the plate is rotated at the proper rate, the same set of crystals gives the maxima for different orders of reflection from a particular crystal plane. For comparison of the intensities of different orders of reflection from the same set of planes, a random orientation of crystals is not required, but this would be essential if we wished to compare the intensity of reflections from different crystal planes.

An account is given in the present Paper of the results obtained by the above method with the two compounds  $\text{Mg}_2\text{Si}$ \* and  $\text{AlSb}$ .

(1)  $\text{Mg}_2\text{Si}$ .—A specimen of the compound  $\text{Mg}_2\text{Si}$  yielded a spectrum, the lines in which were found to be characteristic of the reflections from a face centred cube of side 6.391 Å. The observations are given in Table I, and the full spectrum is plotted in Fig. 1.

TABLE I.— $\text{Mg}_2\text{Si}$ . Face Centred Cube.

Intensity.	Form.	$2\theta$	$\sin \theta$	$a$
0.8	111 $\beta$	9.7	0.0846	6.483 Å
2.4	111 $\alpha$	10.9	0.0950	6.454
0.4	200 $\alpha$	12.3	0.1072	6.588
7.5	220 $\beta$	16.0	0.1392	6.400
17.0	220 $\alpha$	18.05	0.1570	6.375
4.0	311 $\alpha$	21.3	0.1849	6.349
0.6	400 $\beta$	22.8	0.1977	6.372
0.5	331 $\beta$	24.5	0.2122	6.470
2.7	400 $\alpha$	25.6	0.2215	6.390
5.0	331 $\alpha$	27.9	0.2410	6.401
4.6	422 $\alpha$	31.5	0.2714	6.389
0.8	$\left\{ \begin{matrix} 333 \\ 511 \end{matrix} \right\} \alpha$	33.6	0.2890	6.377
2.6	440 $\alpha$	36.5	0.3131	6.395
0.8	531 $\alpha$	38.7	0.3313	6.339
2.2	620 $\alpha$	41.3	0.3524	6.352
				Mean $a = 6.391 \text{ Å}$

$$M = 76.92 \times \frac{1.663}{1.008} \times 10^{-24} \text{ grms.}$$

$$n = 4$$

$$\rho = \frac{4M}{a^3} = 1.94 \text{ grms./c.c.}$$

$$\rho(\text{obsd.}) = 1.88 \text{ grms./c.c.}$$

The observed density can be accounted for on the assumption that there are four molecules associated with each unit cube. This requires a density of 1.94 grms./c.c., while the observed value was 1.88 grms./c.c. The specimen, however, was porous and impure, containing an excess of magnesium. The

\* Although silicon is a non-metal it forms compounds of such distinctly metallic character that it may be grouped for the present purpose with the metals.



discrepancy of 3 per cent. between the observed and calculated density may be due to this cause, an excess of 3 per cent. magnesium being sufficient to account for the difference. We may thus place silicon atoms at the corners and face centres of a cube of side  $6.391 \text{ \AA}$  and have to dispose of eight magnesium atoms in each cube in such a manner as to account for the observed intensities. The most symmetrical arrangement is to suppose the magnesium atoms to be situated at the corners of a cube of side  $\frac{6.391}{2} = 3.19 \text{ \AA}$ , the atoms being situated at the centres of the eight small cubes into which the cube of side  $6.391 \text{ \AA}$  may be divided.

In such an arrangement the (110) planes would all be identical, containing magnesium and silicon atoms in the ratio 2 : 1, and the reflections would be normal. This is approximately the case, and they are also very intense. The (100) spacing would be halved and, accordingly, the spectra of odd order would be weak. The first order was very weak, and the third was not detected. Alternate planes of Si and  $\text{Mg}_2$  would give theoretical intensities  $I_1 : I_2 : I_3 = 0.7 : 3 : 0$ , while the observed

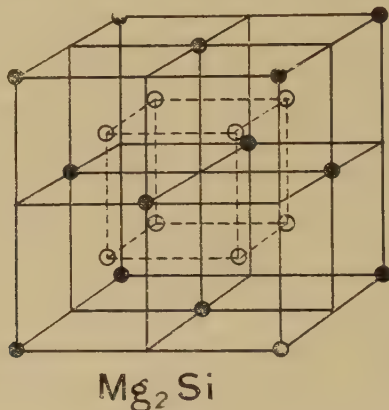


FIG. 2.

intensities are in the ratio 0.5 : 3 : 0. The (111) planes are also abnormal. The distance between successive planes of silicon atoms is divided by two planes of magnesium atoms, dividing the spacing in the ratio of 1 : 3 and 3 : 1. The second-order reflection should theoretically be about one-fifth as intense as the first, while the third-order reflection would be too weak to detect. The second order was, in fact, not detected. The absence of reflection from the (210) planes is also to be expected from the above structure.

The intensities of the reflections from the different faces are, therefore, in general accord with the theoretical values calculated on the assumption that the silicon atoms are arranged on a face-centred cubic lattice with magnesium atoms at the centres of the eight small cubes into which the unit cube of the face-centred lattice may be divided. A unit cell of the structure is shown in Fig. 2.

The closest distance of approach of magnesium and silicon atoms is  $6.39 \times \frac{\sqrt{3}}{4} = 2.77 \text{ \AA}$ . The "radius" of the magnesium atoms is  $1.58 \text{ \AA}$  and that of

the silicon atom  $1.18\text{\AA}$ , or the sum of their radii is  $2.76$ . The atoms thus do not approach one another any more closely than they do in the pure metals.

The magnesium atoms are assembled on a simple cubic lattice of side  $3.19\text{\AA}$  in the compound, while in the pure metal they behave as spheres of diameter  $3.17\text{\AA}$ , the lattice being close-packed hexagonal of axial ratio  $1.63$ . The simple cubic lattice found in the compound is very nearly of the dimension required by spheres of this diameter. The presence of the magnesium atoms causes a separation of the silicon atoms; whereas in pure silicon the nearest distance of approach is  $2.36\text{\AA}$ , in the compound  $\text{Mg}_2\text{Si}$  the smallest distance between two silicon atoms is  $4.5\text{\AA}$ , or nearly double that in the pure element.

(2) Al Sb.—The spectrum obtained with the compound Al Sb is shown in Fig. 3. Twenty peaks were located and were found to correspond to the lines of a

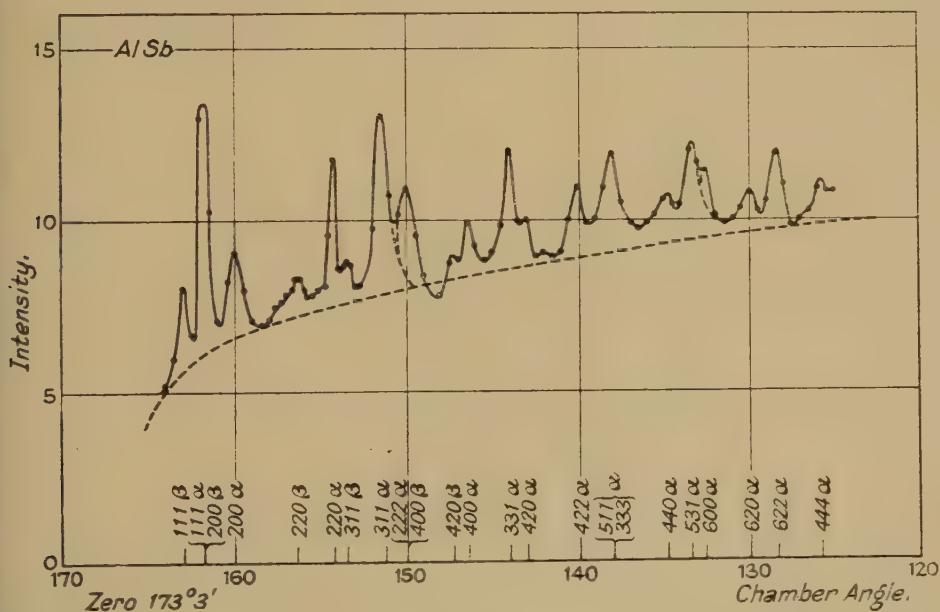


FIG. 3.

face-centred cubic lattice of side  $6.126\text{\AA}$ , as shown in Table II. To account for the observed density four molecules must be associated with each cube. In order to fix the positions of the atoms in the lattice, we have the intensities of the first four orders of the 111 reflections, the first three orders of 100 and the first two orders of 110, as follows:—

		$I_1$	$I_2$	$I_3$	$I_4$
111α } ...	...	7.3	2.6	2.5	1.1
200β } ...	...	2.5	1.7	0.8	...
200α } ...	...	4.1	1.1	...	...
220α } ...	...				



TABLE II.—Al Sb. *Face Centred Cube.*

Intensity.	Form.	$2\theta$	$\sin \theta$	$a$
2.5	111 $\beta$	10.3	0.0898	6.061 $\text{\AA}$
7.3	$\left\{ \begin{smallmatrix} 111 \\ 200 \end{smallmatrix} \right\} \alpha$	11.5	0.1002	6.151
2.5	200 $\alpha$	13.2	0.1149	6.186
1.0	220 $\beta$	16.9	0.1470	6.050
4.1	220 $\alpha$	19.2	0.1668	6.167
1.2	311 $\beta$	19.8	0.1719	6.067
5.0	311 $\alpha$	22.1	0.1917	6.151
2.6	$\left\{ \begin{smallmatrix} 222 \\ 400 \end{smallmatrix} \right\} \alpha$	23.4	0.2028	$\left\{ \begin{smallmatrix} 6.070 \\ 6.121 \end{smallmatrix} \right\}$
0.7	420 $\beta$	26.0	0.2250	6.250
1.7	400 $\alpha$	26.8	0.2317	6.137
3.5	331 $\alpha$	29.3	0.2529	6.128
1.3	420 $\alpha$	30.2	0.2605	6.102
1.9	422 $\alpha$	33.2	0.2875	6.095
2.5	$\left\{ \begin{smallmatrix} 511 \\ 333 \end{smallmatrix} \right\} \alpha$	35.2	0.3024	6.106
1.1	440 $\alpha$	38.3	0.3280	6.130
2.6	531 $\alpha$	39.9	0.3412	6.162
0.8	600 $\alpha$	40.6	0.3469	6.148
0.9	620 $\alpha$	43.3	0.3689	6.094
2.1	622 $\alpha$	45.0	0.3827	6.160
1.1	444 $\alpha$	47.5	0.4027	6.115

$$a = 6.126 \times 10^{-8} \text{ cm.}$$

$$M = 147.3 \times \frac{1.663}{1.008} \times 10^{-24} \text{ grms.}$$

$$\rho(\text{obsd.}) = 4.34 \text{ grms./c.c.}$$

$$n = \frac{a^3 \rho}{M} = 4.10$$

The intensities of the observed orders of the (110) reflections are in the normal ratio of 3 : 1, so that spacings in this direction are as in a face centred cube, successive planes being identical. The ratio of the intensity of the  $\beta$  radiation to the  $\alpha$  radiation is approximately as 1 : 3, so subtracting one third of the intensities of 100 $\alpha$  from the figures in the first line, we have for the 111 planes—

$$I_1 : I_2 : I_3 : I_4 = 6.5 : 2.0 : 2.2 : 0.9.$$

The last figure is estimated on the assumption that  $I_4$  for (100) is approximately one-tenth of  $I_2$ . Remembering that the spacing of the (511) planes is one-third of the spacing of the (111) planes, the apparently great intensity of the (333) reflection is accounted for. The intensities of the (100) reflections are abnormal—the first order is not interfered with by reflection from other planes, the third order falls very close to the position of 444 $\beta$  and the figure 0.8 may, therefore, be about 30 per cent. too large, while the second order is to some extent interfered with by the  $\beta$  reflection from the (210) planes.

If we assume that the antimony atoms are situated on a face centred cube and

that the (100) spacings are halved by planes of aluminium atoms, calculation shows that the (100) intensities to be expected are as 10 : 7·5 : 1. The observed intensities are as 10 : 7 : 3, which is sufficiently good agreement to justify the assumptions made as to the position of the atoms. A structure similar to that of zinc blend (a diamond lattice), with the aluminium atoms arranged on a face centred lattice displaced one quarter of its length along the cube diagonal, would require the intensities of the (111) reflections to be in the ratio 60 : 10 : 6 : 6, while the observed intensities are as 60 : 18 : 20 : 8. The relatively great intensity of the third order reflection is due to the superposition of the reflection from the 511 planes, as already stated. The discrepancy between the observed and calculated intensities of the second order reflections may be due to error in estimating the intensity of the (222) line, as it falls rather near to the comparatively intense reflection from (311). The figure 18 above may quite well be as low as 15, so that the differences between the observed and calculated values do not make the structure suggested impossible.

The evidence indicates that the compound consists of a face centred lattice of

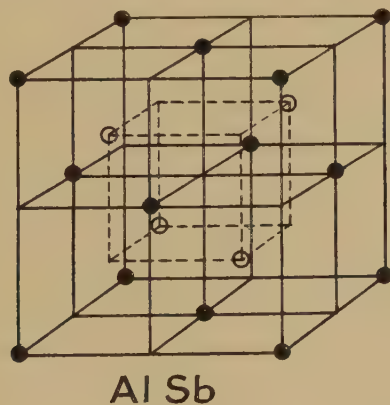


FIG. 4.

antimony of side 6·126 Å intermeshed with an identical lattice of aluminium—the latter having one of its corners situated at a point which divides the cube diagonal in the ratio 1 : 3. A unit cell of the material is shown in Fig. 4.

Antimony is built on two intermeshed face centred rhombohedral lattices\* of side 6·20 Å, the angle between the edges of the rhomb being 86° 58'. A corner of one rhomb is situated very nearly at the centre of the other rhomb, so that the structure approximates to a simple rhomb of side 3·10 Å. In the compound Al Sb the side of the face centred cube is 6·126 Å, so that the antimony lattice contracts slightly, and the angle increases by 3° 2' to 90°. One of the antimony lattices is now replaced by a lattice of aluminium, and the construction point moved along the cubic diagonal towards one corner of the cube. Thus the forces between the atoms of antimony and aluminium are such as to produce a displacement of one lattice relative to the other. The forces involved are, in fact, so great that the com-

\* James and Tunstall, *Phil. Mag.*, p. 233, August (1920).



pound Al Sb is capable of existing in the liquid state,\* and does not melt until a temperature of over 900°C. has been reached, while antimony melts at 630°C. In pure antimony the closest distance of approach of atoms is 2.87 Å. In the compound Al Sb antimony and aluminium atoms are separated by 2.65 Å. The diameter of the aluminium atom is 2.86 Å, so that the aluminium and antimony atoms approach one another closer than in the pure metals.

A further point of interest is that this compound is one of those whose molecular volume is greater than the sum of the atomic volumes of its constituent elements. The sum of the atomic volumes of Al and Sb is 46.3 Å<sup>3</sup>, whilst the molecular volume of the compound Al Sb is 57.5 Å<sup>3</sup>, an increase of about 24 per cent., which agrees with the value obtained by direct measurement.† Now the antimony lattice is practically unchanged in volume by the substitution of aluminium atoms for one set of antimony atoms, so that the whole increase of molecular volume is due to the fact that the aluminium atoms are constrained to lie on a face centred lattice of side 6.126 Å instead of on one of side 4.05 Å, as found in the pure metal, the molecular volume of the compound being 4 per cent. less than twice the atomic volume of antimony.

In conclusion, we wish to express our indebtedness to Miss G. W. Ford and Miss M. L. V. Gayler, of the Metallurgy Department, for the preparation of the specimens.

\* Desch., *Intermetallic Compounds*, p. 80, Longmans Green (1914).

† See Desch., *ibid.*, p. 41.

XXIV.—ON A METHOD FOR THE SYNCHRONOUS AND INSTANTANEOUS ILLUMINATION OF OBJECTS ROTATING OR VIBRATING AT VERY HIGH SPEEDS.

By G. E. BAIRSTO, *D.Sc., D.Eng., F.Inst.P.*, of the Royal Aircraft Establishment.

*Received April 8, 1924.*

ABSTRACT.

A method is described for the instantaneous and synchronous illumination of very rapidly rotating or synchronously vibrating objects.

It is capable of giving instantaneous photographic records, and gives a precision of the order of half a micro-second. It is much more precise and able to give a more intense spark than any contact breaker and coil method.

I. INTRODUCTION.

THE method described in the present report was devised in order to enable the Aerodynamics Department of the R.A.E. to take instantaneous photographs of model propellers running at speeds up to 10,000 revs. per min. As the method is capable of application in many other cases of mechanisms rotating or vibrating at very high speeds it is thought that a full description may be of general interest.

The requirements to be met were as follows: Illumination of the propeller running at 10,000 revs. per min to be accurate in its timing to  $\frac{1}{4}$  of a degree on the shaft and to last no longer than  $1/10$  of a degree. This means that each illumination must occur synchronously with an error not greater than  $4 \times 10^{-6}$  sec., and last no longer than about  $2 \times 10^{-6}$  sec.

II. DESCRIPTION OF THE METHOD.

It was first thought that a contact breaker of the light type used in magnetos, in conjunction with a coil and spark gap, might be arranged to give the required results.

With care and accuracy of fitting, such a contact breaker can be made to give a synchronous timing of sparks accurate to 0.1 degree when running at speeds of the order of 5,000 revs. per min., and it was considered that with special care in construction it could be made to be as accurate at 10,000 revs. per min. Difficulty was, however, experienced in getting a contact breaker spring to withstand the great speed of the blows and at the same time give accurate results; the springs broke after only quite a short run.

Another difficulty in using a coil and contact breaker is that the circuit has its own natural time constant, so that the spark occurs a certain definite time later than the break. This corresponds in small coils or magneto coils to about  $1^\circ$  at 2,000 revs. per min. This entails a very large correction which is very troublesome to apply, as it is not constant, but depends upon the relation of the maximum voltage given by the coil to the spark voltage of spark gap. It was also apparent that a small coil of the type referred to above would not give enough power, and since the time



constant of a larger coil would be still greater, it was decided to abandon any method using coils and use a pure capacity spark.

The scheme in its final form is shown in Fig. 1. A transformer giving a secondary voltage of 20,000 volts is arranged in series with a thermionic valve and a spark gap  $G_1$ , of which one electrode rotates in synchronism with the rotating object, and the other electrode is stationary. In parallel with  $G_1$  is a second spark gap  $G_2$  and a condenser  $C$  in series with it.  $G_2$  is used to illuminate the moving object. The apparatus works as follows: Current from the transformer rectified by the valve charges the condenser  $C$  until the voltage across  $G_1$  reaches its spark voltage. This discharges as soon as the two electrodes of the gap come into line. The condenser then discharges through  $G_2$  and illuminates the rotating object. In order that  $C$  may be charged quickly enough, a high resistance  $R$  is inserted across the spark gap  $G_2$ , otherwise the insulation resistance of  $G_2$  might be so high that enough current could not leak through it in the short interval of one revolution of the rotating object. When  $C$  discharges it will prefer to jump  $G_2$  instead of passing through  $R$ , providing this is large enough.

In order to obtain synchronism of the sparks two things are essential in the

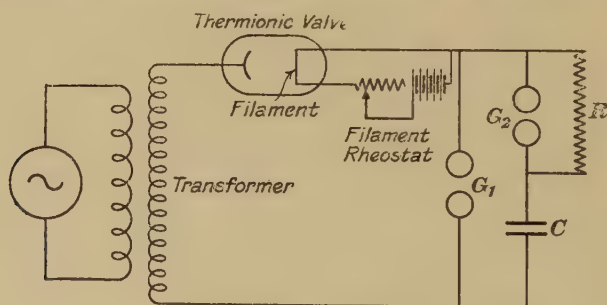


FIG. 1.

spark gap  $G_1$ . First, individual sparks must not vary in the point of the revolution at which they occur. It is obvious, for instance, that no precision could be expected from a spark gap composed of two balls, one stationary and the other rotating, as the spark would take different paths in successive sparks, and the angular error be extremely large. As the gap has to be a certain length, even with pointed electrodes, the irregularity in timing of the spark is very considerable. This difficulty is got over by using two gaps in series, the details of which are shown in Fig. 2. The first gap is formed between a rotating disc fitted with an electrode  $a$ , and a stationary electrode  $b$ ; the second gap is formed between  $(b)$  and another electrode  $c$ . For convenience in adjustment these are arranged at right angles, and  $(c)$  and  $(d)$  are fitted with screw threads. The electrodes  $(a)$  and  $(b)$  are composed of steel, and take the form of cylinders about 3 mm. long and 0.5 mm. diameter, rounded off to a radius of 0.25 mm. The gap  $b-c$  is formed between rods 3 mm. diameter,  $c$  being rounded off to a diameter of the same amount. The gap between  $(a)$  and  $(b)$  is kept as small as possible, being about 0.1 mm. long, while the gap between  $(b)$  and  $(c)$  is proportioned to the spark voltage, and is about 3.5 mm.

The small size of the elements of the first gap give the necessary precision to the timing of the spark, whilst the second gap ensures a high enough working voltage.

The second requirement of  $G_1$  is that it shall discharge immediately the voltage rises to that value which for a statically applied voltage would cause a spark discharge—i.e., there must be no "time lag" in the spark discharge. This is overcome by using a small insulated point ( $d$ ) close to ( $c$ ), so that a small capacity spark passes between them before the spark at the main gap passes. This auxiliary spark provides a continuous source of ionisation and ensures that ions are always favourably placed in the main gap for the starting of the discharge. By this means time lag in the passage of the spark is avoided.

### III. CONDITIONS NECESSARY FOR BEST PERFORMANCE.

There are several conditions which it is necessary should be satisfied if the best results are to be obtained. These are:—

(a) The frequency of the source of alternating potential must be greater than that of the rotating object. If this is not so, then illumination will not occur every

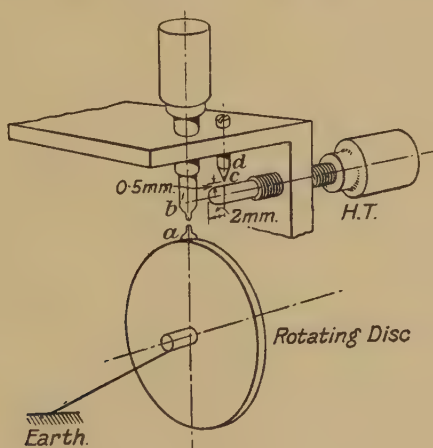


FIG. 2.

revolution. In the case mentioned in the beginning of the paper the frequency of the propeller was 166 cycles per second, and an alternator giving current at 500 ~ per second was used. Alternatively, if such an alternator is not available, a large induction coil with a very rapid and steady break could be used. If only commercial power frequencies are available, then it would be necessary to feed the rectified current into a large condenser and draw on this to supply the spark circuit.

(b)  $R$  should be small enough to allow  $C$  to be charged to the full potential during the interval of one revolution. Assume for simplicity that the frequency of the disc is equal to that of the supply voltage; then, if the rectified sine wave is given by  $e = E \sin pt$ , the current flowing for each charge is given by

$$i = \frac{E}{Z} \sin (pt + \phi) - \frac{E}{Z} \sin \phi e^{-\frac{t}{CR}}$$

where

$$Z = \sqrt{R^2 + \frac{1}{C^2 p^2}}$$

and

$$\cot \phi = CR.$$



Then it can be shown that the condenser voltage, which has its maximum value approximately when  $pt = \frac{\pi}{2}$ , is given by:—

$$[e_{\text{cond}}]_{\text{max.}} = E \left[ 1 - \frac{R^2}{Z^2} + \frac{R}{CpZ^2} e^{-\frac{\pi}{2CRp}} \right]$$

Since  $Cp$  is in practice much greater than  $R$ , this reduces to

$$[E_{\text{cond}}]_{\text{max.}} \doteq E \left[ 1 - R^2 C^2 p^2 + RCp e^{-\frac{\pi}{2CRp}} \right]$$

In order for this expression to be practically constant for any variation in the speed, i.e., say  $[E_{\text{cond}}]_{\text{max.}}$  to be not less than  $0.99E$ , then  $RCp < 0.1$ .

(c) The rectifying valve must be capable of passing enough current to charge the condenser  $C$  up to the required voltage for each spark.

If  $N$  = number of sparks per second, and  $V$  = spark voltage, then  $i = NCV$ .

With a valve passing 100 milliamperes,  $N=200$ , and  $V=20,000$  volts, the maximum permissible capacity  $= 0.025 \mu F$ .

(d)  $R$  must be large enough to prevent the charge of  $C$  falling to an appreciable extent during the interval which corresponds to the slight irregular time lag of the spark gap  $G_2$ .

The e.m.f. of the condenser after discharge of  $G_1$  is given by:—

$$e = E \cdot e^{-\frac{t}{CR}}$$

$e$  should not fall below about  $0.99E$  before the spark passes,

therefore

$$\frac{t}{CR} < 0.01$$

It was found that the irregular timing of the spark was of the order of  $0.5 \times 10^{-6}$ .  
From (1) and (2) we get

$$CR < 10^{-4}$$

and

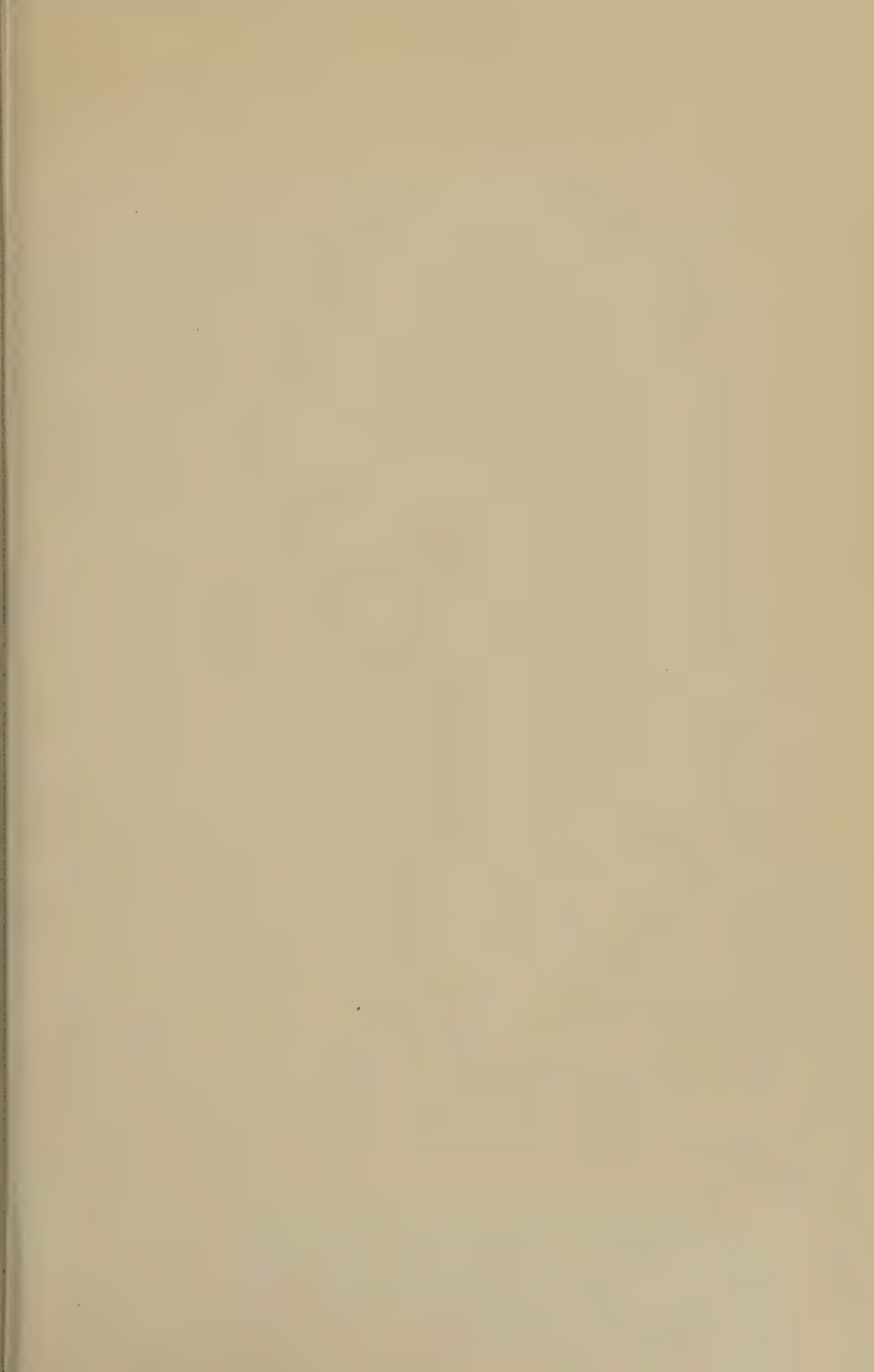
$$CR > 5 \times 10^{-5}$$

For the purpose of the original experiments a capacity of  $1,000 \mu\mu F$  was found to give ample illumination. The resistance  $R$  therefore should be between 50,000 and 100,000 ohms. Experimentally using a disc speed of 166 cycles per second and an alternator frequency of  $500\omega$  per second, the best value was found to be about 100,000 ohms.

#### IV. ACTUAL RESULTS OBTAINED WITH THE APPARATUS.

In order to obtain some idea of the precision\* of the illumination, a photograph was taken of a rotating disc, 8 in. diameter, fitted with a bright nickel-plated round-steel point 1 mm. diameter. Light reflected from this point was received on the photographic plate and an enlargement ( $\times 10$ ) made of the plate. The extent to

\* By this is meant the combination of the departure from synchronism of the spark, and the total duration of the spark.



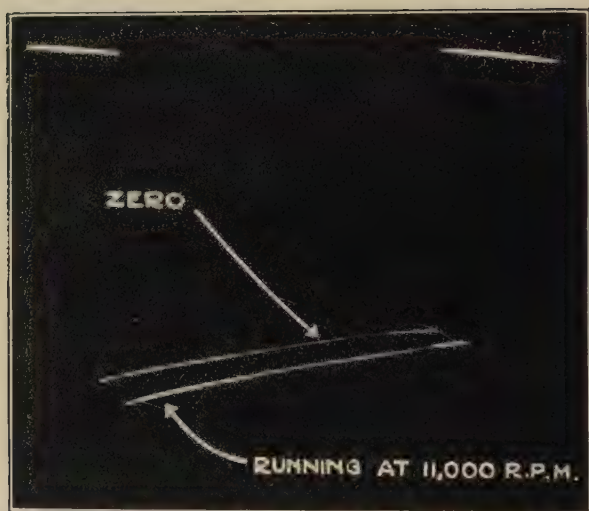


FIG. 3.—PHOTOGRAPH OF SILK THREAD ON BLADE OF PROPELLER.



which the point elongated when rotated at 10,000 revs. per min. was measured and found to correspond to an angular irregularity in the synchronism of the spark of 0.033 of a degree, or to an irregularity in time of 0.55 of a micro-second, which is more than five times as good as was required for the original experiments.

Fig. 3 shows a full-size photograph of a thin silk thread fixed to the end of a blade of model propeller with whitened size. Two exposures were taken, one with the propeller at rest, and one when it was running at a speed of 11,000 revs. per min.

The second line is very little thicker than the first one. It is quite definite and was amply clear enough for the purpose, i.e., the measurement of the angular twist of the blades at very high speeds. The applications of this method to any kind of rotating object or synchronous motion are obvious. It has the advantages of enabling photographic records to be taken easily and instantaneously, as considerable energy can be dissipated in the spark in a form which gives rise to light of an extremely actinic nature.

#### V. CONCLUSIONS.

(a) The method described is capable of giving synchronous and instantaneous illumination of objects rotating at very high speeds, or of synchronous vibrations, with a precision of the order of half a micro-second.

(b) It is capable of giving instantaneous photographic records of such objects.

(c) It is much more precise and able to give a more intense spark than any coil and contact-breaker method.

The author desires to acknowledge his thanks to the Director of Research, Air Ministry, for permission to publish this Paper, and to Mr. G. P. Douglas, B.Sc., who took the photographs.

#### DISCUSSION.

Dr. E. H. RAYNER expressed great appreciation of the accurate and simple method described, which should be very generally useful as intermittent illumination is often required at speeds lower than that contemplated in the Paper. Apparently spark illumination is extremely effective actinically, and it fortunately happens that air is the most effective gas for the purpose. He would suggest that it would be better to earth the filament rather than the anode in Fig. 1. In Fig. 2, should not the parts *b* and *d* be shown as connected electrically, since the whole of the bracket supporting them was stated to be insulative? Was there any reason why ultra-violet light should not be used for ionising the spark gap instead of the pilot gap *cd*?

Mr. J. GUILD said that being interested in a similar problem, though in relation to lower speeds, he was not surprised to notice that the author had early decided that contact-breakers were unsatisfactory for any but low speeds. For visual, as opposed to photographic, methods spark illumination is unsuitable; in such cases is there any reason why a neon tube in series with the gap  $G_1$  should not be employed, the gap  $G_2$  being in that case abolished? And could not the generator and transformer be replaced by an induction coil?

Mr. F. E. SMITH said that for examining rotating objects it is the practice at the Admiralty to use the Elverson oscilloscope, which comprises a neon lamp illuminated intermittently in synchronism with the rotation. In this way the object can be made to appear stationary, while by making the illumination period slightly longer than the revolution period an appearance of slow rotation is obtained.

The AUTHOR, in reply to Dr. Rayner, said that the top, not the bottom line of the diagram, Fig. 1, was supposed to be earthed, but he had not troubled to show this. The parts *b*, *d* are

intentionally insulated from one another ; the effect of the mutual capacities is to ensure that a pilot spark shall pass from the point *d*, although this is completely insulated by the bracket. He had not found in practice that ultra-violet light was so easy to apply or so certain in its effect as the pilot spark. In reply to Mr. Guild, he had no doubt that a neon lamp would work as suggested, but in the present instance he had required photographs, which are unobtainable with the neon lamp unless special plates are used. Further, a certain expenditure of energy is required in each flash in order to obtain sufficient illumination, and in arranging for this expenditure it is probable that precision would have to be sacrificed. In reply to Mr. Smith, he said he was familiar with the Elverson oscilloscope and had, in fact, advised the inventor when it was being designed ; but though admirable at speeds up to 2,000 revs. per min., it failed completely at the speeds contemplated in the Paper.

XXV.—ABSORPTION AND SCATTERING OF GAMMA-RAYS.

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*Received April 19, 1924.*

ABSTRACT.

(1) The absorption and scattering of gamma-rays from radium filtered through 23 mm. of lead have been measured in magnesium, aluminium, zinc, tin and lead.

(2) *On the assumption that the mean effective wave-length of the radiation employed is 0.021 Å, the experimental results are consistent with the following statements:—*

(i) When gamma-rays traverse matter, the characteristic radiations of the absorbing medium are excited.

(ii) The atomic fluorescent absorption coefficient of gamma-rays depends upon the wave-length of the incident radiation and the atomic number of the absorber according to the law  $\frac{\tau}{\rho} \cdot \omega = K\lambda^3 N^4$ , which holds for X-rays.

(iii) The radiations which accompany this fluorescent absorption are the characteristic radiations of the *K, L, M,...* series of the absorbing elements.

(iv) The absorption of gamma-rays in light elements is due almost entirely to scattering.

(v) The pure atomic scattering absorption coefficient is proportional to the atomic number of the absorber.

(vi) In addition to fluorescent and scattering absorption, a true absorption exists, the atomic coefficient of which is proportional to the atomic number.

(3) Compton's formulæ, deduced from his Quantum Theory of Scattering, would account for the experimental results if the wave-length of the incident radiation were 0.020 Å. Jauncey's formulæ would require the wave-length to be 0.029 Å.

IN an investigation on the scattering and absorption of gamma-rays, Ishino\* found that the greater part of the total absorption in light elements was due to scattering, and that there existed also in all the elements examined a true absorption which was approximately directly proportional to the atomic number of the absorbing element. It is of interest to enquire into the nature of this true absorption and to obtain, if possible, further data concerning the whole phenomenon of the absorption by matter of radiation of short wave-length. Such data have an important bearing upon the theories of electromagnetic radiation and of atomic structure.

A brief reference to some preliminary measurements on the absorption of gamma-rays from radium and the conclusions drawn therefrom was made in the annual report of the National Physical Laboratory for 1921. The main result arrived at was that part of the absorption, especially in the heavy elements, was due to the excitation of the characteristic radiation of the absorber. Further measurements have confirmed this conclusion, and, in addition, data have been obtained concerning the scattering of the radiation.

\* Ishino, *Phil. Mag.*, 33, 140 (1917).



## DESCRIPTION OF APPARATUS.

The apparatus initially employed differed from that used by Ishino in that a rectangular chamber, the electrode of which was connected to a Wilson tilted electro-scope, measured the ionisation current instead of an ordinary electro-scope. With this apparatus results of the same order of magnitude as those recorded by Ishino were obtained for the absorption and scattering, although our measurements gave lower values for what he termed the true absorption. The apparatus was capable of a fair degree of accuracy, but it was considered that the form of the apparatus was such that the results obtained with it could not be easily interpreted, the chief drawback being that the rectangular chamber did not measure the ionisation produced by radiation emitted in all directions from the radiator. The apparatus finally adopted is shown in Fig. 1. Here the rectangular ionisation chamber is replaced by a spherical one, the insulated electrode of which is connected through an earthing key to a pair of the quadrants of a Dolezalek electrometer having a comparatively low sensitivity—about 1,000 divisions per volt on a scale placed about a metre in front of the instrument.

The results of different observers on the absorption of gamma-rays have shown

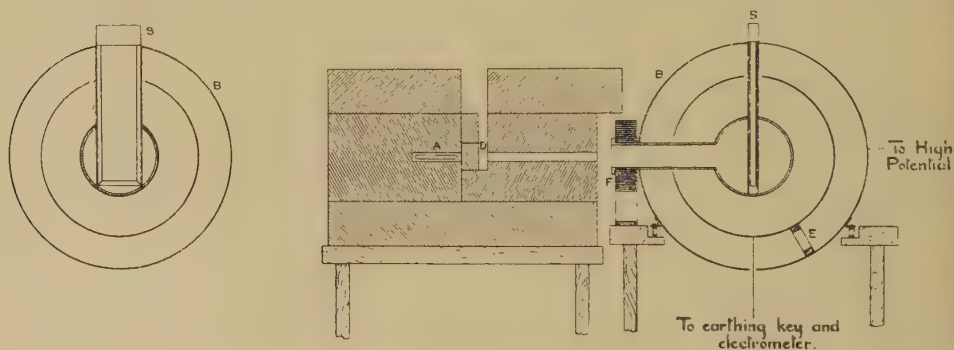


FIG. 1.

variation according to the disposition of the apparatus employed. This is probably to be accounted for by the different amount of scattered radiation which entered the ionisation chamber. In view of this, it was deemed advisable to give the following details concerning the experimental arrangement adopted here, so that, if desired, the experiments can be repeated by other observers.

The source of gamma radiation is a tube of radium 40.3 mm. in length and 6.4 mm. in diameter, containing the equivalent of 172 milligrammes of radium element in radio-active equilibrium. It is embedded at *A* (Fig. 1) in a lead block measuring 20 by 20 by 30 cm., which is made up in sections overlapping each other and fitting closely together. The mean distance of the tube from the end of the block remote from the ionisation chamber is 12.5 cm. The gamma-rays are filtered through 23 mm. of lead before reaching a rectangular hole 1 cm. square and 12 cm. long, in which the radiation is formed into a narrow beam. This hole is accurately machined, and a rod of lead fitting it closely can be inserted to block it up. In front of the 23 mm. lead filter and extending to the top of the lead block, is a space *D*, about 6 mm. wide and 3 cm. broad, into which the absorbing screens are placed.

The spherical ionisation chamber *B* is constructed from three copper spheres of diameters 9 cm., 18 cm., and 25.5 cm. respectively. The inner sphere is lined with lead 2.8 mm. thick, and is connected to the outer sphere. It is fixed to a brass tube of internal diameter 2.5 cm., held in an insulated metal clamp at *F*. The middle sphere which constitutes the insulated electrode is held in position by a rod of ebonite *E* into which a wide sulphur ring is inserted to increase the insulation. This is connected to the electrometer by means of an earth-guarded wire passing through a hole in the outer sphere. The potential of the inner and outer spheres is 240 volts. The absorbing screens are inserted at the centre of the inner sphere by means of a carrier which moves in a slide *S* (8 mm. by 5 cm. in cross-section), extending to the top of the outer sphere. To obtain consistent readings when the screens are in this position, it was found that the portion of the slide between the inner and the outer spheres had to be completely covered in. When a skeleton slide was employed, the readings were unsteady for some time after the screens were lowered into position. The precaution of covering in the slide, as indicated above, remedied this defect.

## METHOD OF PROCEDURE.

The ionisation currents in the spherical chamber were measured when different thicknesses of absorbing screen were placed inside the blocks and inside the spheres respectively. In the first position all the secondary radiation excited in the screens is cut away from the ionisation chamber, whereas in the second position a portion of the secondary radiation enters the chamber. For convenience the intensity of the radiation in the two cases will be denoted by  $I_1$  and  $I_2$ .

The natural leak of the apparatus is an important factor for which allowance must be made. The apparatus is so arranged that the lead blocks can be removed and replaced again in the same position relative to the chamber. This allows the lead rod to be inserted into the rectangular hole defining the beam of radiation, so that a measurement of the natural leak can be made with the radium in position. The leak measured in this way differs appreciably from that observed when the radium is removed to a great distance.

The four metals—aluminium, zinc, tin and lead—were investigated in detail. The procedure in taking readings was to observe the time taken by the electrometer leaf to deflect through a definite angle with no absorbing screen and with increasing thicknesses of absorbing screen within the blocks or within the spheres.

## RESULTS.

The relative ionisation currents observed with different thicknesses of absorbing screen are given in Table I. The figures are corrected for natural leak, and represent the means of a large number of observations.

When the values of  $I_1$  and  $I_2$  are plotted against thickness of absorbing screen, it is found that for the small thicknesses examined the resulting curves are exponential within the error of observation, so that the results may be represented by the formulæ :—

$$I_1 = {}_0I_1 e^{-\mu_1 t} \text{ and } I_2 = {}_0I_2 e^{-\mu_2 t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

TABLE I.

Aluminium.			Zinc.			
Thickness of screen. in cm.	$I_1$ (inside lead block).	$I_2$ (inside spheres).	Thickness of screen. in cm.	$I_1$ .	Thickness of screen. in cm.	$I_2$ .
0.0	100	100	0.0	100	0.0	100
0.105	98.1	99.4	0.05	97.6	0.052	99.1 <sub>5</sub>
0.210	97.3	98.9	0.102	96.2	0.104	98.2
0.315	95.2	97.9	0.204	92.2	0.207	97.1
0.42	93.9	97.7	0.306	88.5	0.310	95.2
0.525	92.3	97.3	0.408	85.1	0.414	94.0
			0.510	81.4	0.517	92.8

Tin.			Lead.			
Thickness of screen. in cm.	$I_1$ .	$I_2$ .	Thickness of screen. in cm.	$I_1$ .	Thickness of screen. in cm.	$I_2$ .
0.0	100	100	0.0	100	0.0	100
0.050	98.1 <sub>5</sub>	99.3	0.053	96.3	0.050	98.9 <sub>5</sub>
0.100	96.3	98.5	0.104	92.8	0.101	97.4
0.200	92.3	97.0 <sub>5</sub>	0.202	85.8	0.198	94.1
0.300	89.5	95.5	0.305	80.4	0.299	92.0
0.400	85.2	94.5	0.403	73.9	0.395	88.9
0.500	82.0	92.7	0.518	68.9	0.497	86.8

where  $t$  is the thickness of the absorbing screen in centimetres. The values of  $\mu_1 t$  and  $\mu_2 t$  calculated from the above formulæ are shown in Figs. 2 and 3\* plotted against the thickness  $t$ . From these straight lines the mean values of  $\mu_1$  and  $\mu_2$  given in Table II. were obtained. The values of the mass and atomic absorption coefficients are also included for reference in the same table. The value of  $\mu_1$  for magnesium appearing in this table was obtained with a single sample of the material.

TABLE II.

Absorbing screen.	At. No. of absorber.	$\mu_1$	$\mu_2$	$\frac{\mu_1}{\rho}$	$\frac{\mu_2}{\rho}$	$\frac{\mu_1}{\rho} \cdot w \times 10^{24}$	$\frac{\mu_2}{\rho} \cdot w \times 10^{24}$
Mg	12	0.098 <sub>5</sub>	...	0.0566	...	2.2	...
Al	13	0.151	0.056	0.0559	0.0207	2.5	0.93
Zn	30	0.403	0.151	0.0567	0.0213	6.1	2.3
Sn	50	0.383	0.148	0.0525	0.0203	10.3	4.0
Pb	82	0.720	0.292	0.0633	0.0257	21.6 <sub>5</sub>	8.8

In previous work on this subject  $\mu_2$  has been taken as a measure of the true absorption coefficient, and has been found to be approximately directly proportional to the atomic number of the absorbing element. It will be observed from Fig. 4, in which the values of the atomic absorption coefficients given in the last two columns of Table II. are plotted against atomic number, that a linear relation does not exist

\* The values for zinc are not shown in these figures. They are omitted to avoid confusion, as they are very nearly the same as those for tin.



between  $\mu_2$  and  $N$ . Neither is this true for  $\mu_1$  and  $N$ , the departure from proportionality being even more marked in this case. To explain these results, it is evident that we must take a different view of the absorption phenomenon of gamma radiation from that previously adopted.

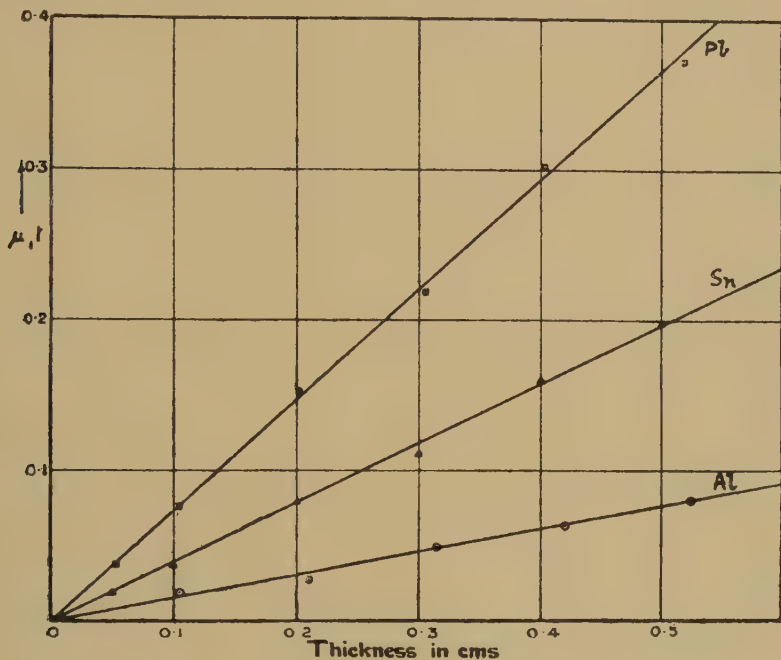


FIG. 2.

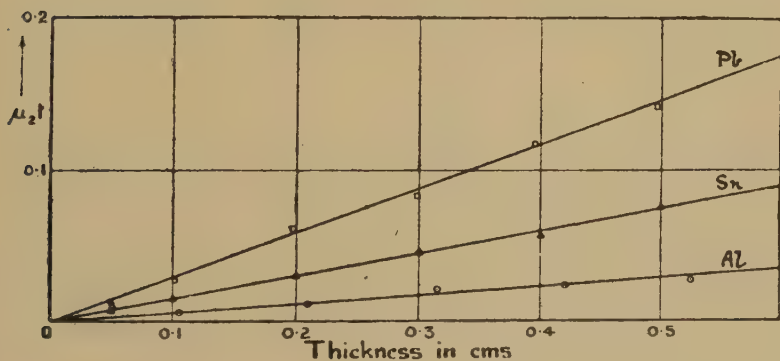


FIG. 3.

#### THEORETICAL.

It has been fairly well established, both experimentally and on theoretical grounds, that for X-rays the atomic fluorescent absorption coefficient is proportional to  $\lambda^3 N^4$  where  $\lambda$  is the wave-length of the incident radiation and  $N$  the atomic number

of the absorber. This law has not so far been extended to include wave-lengths as short as those of gamma-rays.

A good deal of doubt exists as to whether the phenomenon of scattering observed with X-rays is of the same nature as that observed with gamma-rays. Experiment appears to show that the scattering laws governing the former do not hold for the latter. Thomson's classical theory, which explains satisfactorily the scattering of X-rays by light elements when radiations of moderate wave-lengths are employed, fails to account for the results observed with gamma-rays. For instance, the total mass absorption coefficient for hard gamma-rays has been found to be of the order of 0.05, whilst the minimum value of the scattering alone predicted by the classical theory is 0.2. The theory also fails to account for the excess amount of scattered radiation on the emergent side of a screen irradiated by X or  $\gamma$  radiation.

Recently Compton\* has put forward a quantum theory of scattering, which appears to explain the above facts satisfactorily. It is assumed that when an X-ray or a gamma-ray quantum is scattered, it spends all its energy and momentum upon a single electron which scatters the quantum in some particular direction. If the direction of propagation of the scattered-ray is different from that of the incident ray, a change in momentum of the gamma-ray quantum takes place which results in the recoil of the scattering electron. The energy of the scattered quantum will, therefore, be less than that of the primary quantum by the kinetic energy of recoil of the scattering electron. In other words, the scattered radiation will have a longer wave-length than the primary radiation. On this view it is evident that the wave-length of the incident radiation must be less than the distance between two adjacent scattering electrons. If this be not the case, a group of electrons will take part in the scattering and, since their combined mass is comparatively great, no appreciable amount of energy is lost by recoil and the wave-length of the scattered ray will be the same as that of the incident ray. On this theory the total energy removed from the primary beam by scattering is less than that given by Thomson's theory in the ratio of  $1 : (1 + 2\alpha)$  where  $\alpha = h/mc\lambda$ ,  $\lambda$  being the wave-length of the incident radiation,  $h$  Planck's constant, and  $m$ , the mass of the scattering electron. A part of this energy appears as scattered radiation and the remainder is truly absorbed and transformed into the kinetic energy of recoil of the scattering electron.

It will be assumed that gamma-rays lose energy by traversing matter in the following ways: (1) By the emission of corpuscular radiation, which is accompanied by characteristic radiations of the absorbing material, (2) by scattering, i.e., the emission of electromagnetic radiation which may or may not be of the same wave-length as the primary radiation, and (3) by the absorption of energy by electrons either resulting in their emission with the full quantum of energy of the incident radiation but unaccompanied by the emission of characteristic radiation of the traversed material (in which case the electrons emitted would come from the outer layers of the atom), or, appearing as kinetic energy of recoil of the scattering electrons in accordance with Compton's views.

Turning now to our present experiments, the absorption measured when the absorbing screens are situated inside the lead block is an accurate determination of the *total* absorption, including the loss of energy by all the processes above mentioned.

\* A. H. Compton, *Phys. Rev.* 21, 483 (1923). See also *Bull. Nat. Res. Council*, No. 20 (1922).

Curve *B* of Fig. 4, represents the experimental results obtained when the screens are situated at the centre of the spherical ionisation chamber. In this case  $\mu_2 = K\tau + \sigma_a$ , where *K* has the value unity when the characteristic radiation excited in the absorbing screen is so soft that it does not penetrate the wall of the inner sphere of the chamber to any appreciable extent. The wall of the inner sphere is



of lead 2.8 mm. thick, and calculation shows that the amount of the  $K$  characteristic radiation of tin (wave-length  $0.49 \text{ \AA}$ ) which enters the ionisation chamber is negligible so that for tin  $\mu_2 = \tau + \sigma_a$ . This relation also holds for zinc radiation. Taking the values of  $\frac{\tau}{\rho} \cdot w$  for tin and zinc from curve  $A$  and decreasing the ordinates of curve  $B$ , by these amounts, it is found that the resulting points lie very approximately on a straight line through the origin and through the observed value of  $\mu_2$  for aluminium. The difference between the ordinate of this straight line and that of curve  $B$  for lead is, however, 3.2 instead of 5.8, as observed in curve  $A$ . This difference is due to the fact that the characteristic radiation excited in lead penetrates the wall of the inner sphere. Hull found that  $\frac{\mu}{\rho}$  for  $K$  characteristic radiation of lead in lead is 1.51 approximately, so that about 0.7 per cent. of the radiation will be transmitted through the wall of the inner sphere. Correcting for this effect the

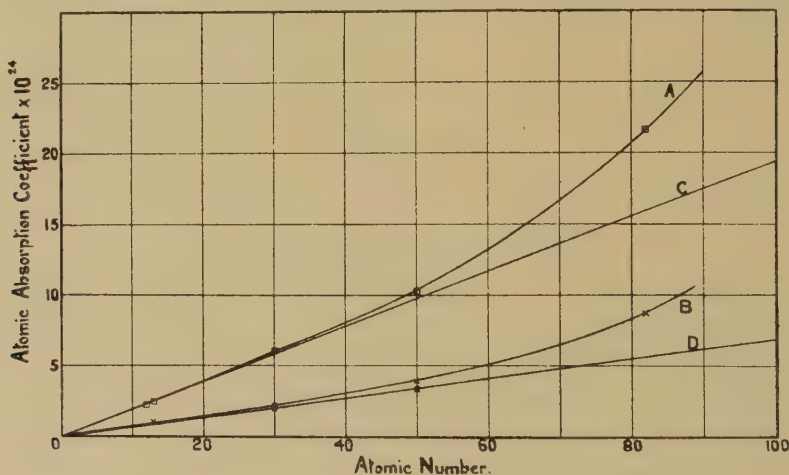


FIG. 4.

difference between the ordinates of the curves  $B$  and  $D$  at the point corresponding to lead becomes equal within the error of measurement to the value of  $\frac{\tau}{\rho} \cdot w$  found for lead from the curves  $A$  and  $C$ .

The straight line  $OD$  (Fig. 4) represents the absorption of energy by electrons according to (3) on p. 361. Its equation is

$$\frac{\sigma_a}{\rho} \cdot w = 0.68 \times 10^{-25} N \quad \dots \dots \dots (5)$$

In the absence of definite data regarding the effective wave-length of gamma radiation from radium filtered through 23 mm. of lead, a satisfactory comparison of the experimental results with the theoretical formulæ for scattering deduced by Compton or with the modified formulæ put forward by Jauncey is impossible. It may, however, be of interest to calculate from these theoretical formulæ what

the mean wave-length should be to account for the scattering observed in the present investigation.

According to Compton's theory, the total scattering is given by the equation

$$\frac{\sigma}{\rho} \cdot w = \frac{\sigma_s + \sigma_a}{\rho} \cdot w = \frac{\sigma_0}{\rho} \cdot \frac{w}{1+2\alpha} = \frac{8\pi}{3} \cdot \frac{Ne^4}{m^2c^4} \cdot \frac{1}{1+2\alpha} \quad (6)$$

where  $\alpha = h/mc\lambda$ ,  $\lambda$  being the wave-length of the incident radiation,  $c$  the velocity of light,  $N$  the atomic number and  $e$  the charge on the scattering electron measured in electrostatic units. This equation gives the value  $1.93 \times 10^{-25}N$  found by experiment for the total scattering when the wave-length is  $0.020\text{\AA}$ .

Similarly, the absorption due to the kinetic energy of recoil of the scattering electron is given by the equation

$$\frac{\sigma^a}{\rho} \cdot w = \frac{\sigma_0}{\rho} \cdot w \cdot \frac{\alpha}{(1+2\alpha)^2} \quad (7)$$

which gives the experimental value  $0.68 \times 10^{-25}N$  when the wave-length is also  $0.020\text{\AA}$ .

A mean wave-length of  $0.020\text{\AA}$  would, therefore, account for the experimental results on Compton's formulæ.

In deducing the above expressions for the intensity of scattered radiation, Compton reasoned from analogy with the Doppler effect in a manner which, as he himself points out, is not rigorous. Jauncey\* has attempted to avoid this uncertainty by assuming that "quanta of X-rays in the form of corpuscles are deflected by the electrons according to a law of force such that, for corpuscles of small momentum (low-frequency quanta), the distribution of the scattered rays is that expressed by the classical theory." He thus obtains a relation between the angle of deflection of the quantum and the distance of its undeviated path from the scattering electron and uses this relation to calculate the intensity of the scattered radiation in the case of high-frequency quanta. His expressions for the scattering differ only by second and higher powers of  $\alpha$  ( $\equiv h/mc\lambda$ ) from those of Compton, and in the region of X-rays ( $\alpha < 0.1$ ) the difference amounts at most only to a few per cent. For gamma-rays of short wave-length, however, the difference is appreciable.

According to Jauncey, the total scattering is given by the equation

$$\frac{\sigma}{\rho} \cdot w = \frac{8\pi}{3} \cdot \frac{Ne^4}{m^2c^4} \cdot \frac{1}{(1+\alpha)^2} \quad (8)$$

which would require the wave-length of the incident radiation to be  $0.029\text{\AA}$  to give the experimental value for the total scattering.

Since the radiation is filtered through 23 mm. of lead, it consists mainly of the gamma radiation emitted by radium C. The highest energy lines so far observed by Ellis† in the  $\beta$ -ray spectrum of radium C correspond to gamma radiations of wave-lengths  $0.025$ ,  $0.021$  and  $0.020\text{\AA}$ . Tuomikoski‡ found that the absorption coefficient of gamma radiation from radium showed little change for thicknesses of lead varying from 22 mm. to 120 mm. Thus, for a range of thickness of lead

\* Jauncey, Phys. Rev., 22, p. 233 (1923).

† C. D. Ellis, Proc. Roy. Soc., A, 101, 6 (1922).

‡ Tuomikoski, Phys. Ztschr., 10, 372 (1909).

from 22 mm. to 54 mm.  $\mu$  is 0.52, and for thicknesses between 54 mm. and 120 mm.  $\mu$  is 0.50. This small change in the value of the absorption coefficient over such a great thickness of material would indicate that if, as is probable, radiations of shorter wave-lengths than those already recorded by Ellis are emitted, they must be of small intensity compared with the three lines mentioned.

Since the line 0.020 Å is the most intense of these three lines, it would not be unreasonable to take 0.021 Å as the mean effective wave-length of the radiation dealt with in the present investigation, and, on this assumption, Compton's theoretical formulæ for the scattering represent the experimental results very satisfactorily.

Returning now to the consideration of the fluorescent absorption, the differences between the ordinates of the curve *A* and the straight line *OC* in Fig. 4 give the values of the atomic fluorescent absorption coefficients. These are set out in column 2 of Table III. The values relative to lead, calculated on the assumption that the atomic fluorescent absorption coefficient is proportional to the fourth power of the atomic number, are given in column 3 of the same table. It will be observed that the observed and calculated values are in tolerable agreement, from which we conclude that the absorption in excess of that due to scattering obeys the same

TABLE III.

Absorber.	$\frac{\tau}{\rho} \cdot w \times 10^{24}$	
	Observed.	Calculated.
Zn	0.2	0.1
Sn	0.7	0.8
Pb	5.8	5.8

relation as that found for X-rays as far as the atomic number of the absorber is concerned. If, as has already been assumed, this fluorescent absorption is associated with the *K*, *L*, *M*, ..... characteristic radiations of the absorbing element, and the absorption coefficient varies as the cube of the wave-length as with X-rays, a knowledge of the absorption coefficient for a radiation of any given wave-length enables us to calculate its value for gamma-rays. This procedure is possible only if the absorption is in the region between the same absorption bands for the wave-lengths considered. The necessary data for the calculation are available for lead and zinc.

Hull and Rice\* found the value of  $\frac{\tau}{\rho}$  in lead for a wave-length 0.122 Å to be 2.88. Hence the atomic fluorescent absorption coefficient in lead for a wave-length 0.021 Å is

$$2.88 \times \frac{207.2 \times 1.664}{1.008} \times \left( \frac{0.021}{0.122} \right)^3 \times 10^{-24} = 5.1 \times 10^{-24},$$

the observed value being  $5.8 \times 10^{-24}$ . The value of  $\frac{\tau}{\rho} \cdot w$  in zinc for radiation of wave-length 0.586 Å measured by Owen† is  $32.8 \times 10^{-24}$ , so that the atomic fluorescent absorption coefficient in zinc for gamma radiation of wave-length 0.021 Å,

\* Hull and Rice, *Phys. Rev.*, **8**, 836 (1916).

† Owen, *Proc. Roy. Soc., A*, **94**, 522 (1918).



calculated as above, is  $0.16 \times 10^{-24}$ , as against the observed value of  $0.2 \times 10^{-24}$  given in Table III. The values of the absorption coefficients of X-rays on which the above calculations are based are those in the *K* region. Considering the wide range of extrapolation, the calculated values show very good agreement with the observed values. These results support the following conclusions:—

When gamma-rays are absorbed by matter, the characteristic radiations of the absorbing medium are excited. The atomic fluorescent absorption coefficient depends upon the wave-length of the incident radiation and atomic number of the absorber according to the law

$$\frac{\tau}{\rho} \cdot w = K \cdot \lambda^3 N^4 . . . . . (9)$$

which holds for X-rays. The radiations which accompany this fluorescent absorption are the characteristic radiations of the  $K, L, M, \dots$  series of the absorbing elements.

The values of the absorption coefficients  $\tau$ ,  $\sigma_s$  and  $\sigma_a$  and the corresponding mass and atomic absorption coefficients derived from the foregoing results are included in Table IV.\*

TABLE IV.—*Absorption Coefficients for Gamma Radiation from Radium Filtered through 23 mm. of Lead.*

Absorber.	$\tau$	$\sigma_e$	$\sigma_a$	$\tau/\rho$	$\sigma_e/\rho$	$\sigma_a/\rho$	$10^{24}$		
							$\tau/\rho \cdot w$	$\sigma_e/\rho \cdot w$	$\sigma_a/\rho \cdot w$
Mg	...	0.06	0.03 <sub>g</sub>	...	0.035	0.020	...	1.4	0.8
Al	...	0.10	0.05	...	0.036	0.020	...	1.6	0.9
Zn	0.013	0.25	0.13	0.0019	0.035	0.019	0.2	3.8	2.0
Sn	0.026	0.23	0.13	0.0036	0.032	0.017	0.7	6.2	3.4
Pb	0.19	0.34	0.19	0.017	0.030	0.016	5.8	10.2	5.6

The fluorescent absorption is a measure of the loss of energy of the primary beam due to the emission of electrons from different energy levels in the atom. This emission of electrons gives rise to characteristic radiations in accordance with Bohr's views. In addition, the dislodged electrons produce secondary radiation by impact with atoms in their paths. It should be noted that these secondary effects are included in the measurement of the fluorescent absorption.

It has been suggested that the coefficient  $\sigma_a$ , which we called the corpuscular absorption coefficient, may possibly be due to the energy absorbed in the emission of electrons from the lowest energy levels in the atom. Since the work done by these electrons in getting free of the atom is negligible, their energy is equal to that of a full quantum of incident radiation in accordance with the relation  $\frac{1}{2}mv^2 = h\nu$ ,  $v$  being the velocity of the expelled electron. The fact that no electrons having a velocity corresponding to the value given by the above relation have been observed in  $\beta$ -ray experiments, disposes of the possibility of the emission of electrons from the outer electronic layers of the atom with the energy of a full quantum. We are

\* The densities throughout have been taken from Kaye and Laby's Tables, 4th Ed., p. 22.

therefore left with the second alternative that  $\sigma_a$  represents the kinetic energy of recoil of the scattering electrons, as put forward by Compton.

There is a good deal of ground for the assumption that the absorption of gamma-rays in light elements is due entirely to scattering. Barkla and White\*, for instance, infer from their work on the absorption and scattering of X-rays of short wave-length that "the absorption of gamma-rays in substances of low atomic weight is almost entirely absorption by scattering, which is proportional to the number of electrons in the substance traversed." That the proportionality between the scattering coefficient and the atomic number extends to the heavy elements departs from the generally accepted view, although, up to the present, the fluorescent absorption and the absorption by scattering for very short wave-length radiation have not been separated in the way attempted here.

The experiments are being extended to include a larger number of the elements.

#### DISCUSSION.

Dr. I. SIMONS, in congratulating the authors on their work, mentioned that they had brought together in their Papers a good deal of information from various sources which had previously been as scattered as the radiation with which it was concerned. The somewhat revolutionary ideas of Compton seemed to be a harking back to the views entertained by Sir Wm. Bragg before the war, but subsequently more or less relinquished by him. There was one point in the Paper as to which he was not quite clear: Compton's theory applies to radiation scattered in a definite direction, so that an angle always enters into his formulæ. No such angle occurred in the authors' formulæ, and as the ionisation chamber was spherical, Compton's equations were not applicable in the form given by him. It might be of interest to add that he had for two years been making experiments on the relation between the true  $\beta$ -particles from atoms and the slow  $\delta$ -particles, the results of which appeared to support Compton's views. He expressed a hope that as the number of transformation coefficients had already reached three no further multiplication of these would be considered necessary in the near future.

Dr. F. L. HOPWOOD said that Compton's theory might throw some light on a subject that was very puzzling to radio-therapeutists, viz., that in the case of very hard rays of which the absorption is small, their biological effect seems out of all proportion to the energy absorbed by the tissue. Had the authors used gases other than air, such as one of the heavy gases, in the ionisation chamber?

Dr. E. A. OWEN (in reply to the discussion) said that the corpuscular absorption coefficient introduced in connection with Compton's theory must be regarded as representing an integration or average value taken over all directions in space. He was much interested in the experiments mentioned by Dr. Simons, as he was becoming more and more convinced that Compton's theory was right. No gases other than air had been used; it would be courting error to work with heavy gases, which with hard radiation might give rise to complicated effects.

\* Barkla and White, *Phil. Mag.* 270 (1917).

## XXVI.—THE FLOW OF COMPRESSIBLE FLUIDS, TREATED DIMENSIONALLY.

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### ABSTRACT.

The method of dimensions treatment that is applicable to the pressure gradient at a point in a system through which non-compressible fluids of finite viscosity are passed, is in this Paper extended by means of the thermo-dynamical equations for gas flow to the case where appreciable changes in density of the fluid occur, but where no heat passes across the walls of the system. The theory is developed in detail only for the case of flow through a straight parallel-walled tube.

The theory is then tested by means of experiments in which water and air at high velocities passed through small tubes. The air in some experiments had a velocity of more than two-thirds of the velocity of sound in the air. The possibility of errors due to moisture, pulsating flow, heat conduction through the walls, and proximity to the entrance to the tube is considered, experimentally and by approximate calculation, and the errors are shown to be small in the present experiments. An error of moderate amount is, however, found, and attributed to the partial neglect of the variation of the variables over the transverse section of the tube. The value of the error varies in the way it might be expected to do, and is of the order that is expected by other considerations.

### I. INTRODUCTION.

THIS Paper consists of a theoretical and experimental investigation of the variation from point to point of the pressure, density, temperature and velocity of a gas moving at speeds comparable with that of sound in the gas, when some of the energy is being converted into heat directly or indirectly on account of viscosity. The Paper deals in detail only with the case of flow through a straight, parallel-walled tube, but the method is capable of extension to other cases. The method of dimensions treatment, such as is applicable to cases when the density of the fluid is almost constant, is here extended with the help of the thermo-dynamical equations for gas flow, in order to obtain a method of relating the cases when the density is sensibly constant (liquid flow, or slow gas flow) with any cases when the density varies appreciably from point to point. The relationship is then tested by means of experiments on the flow of water and air at high velocities through the same tube.

### II. DIMENSIONAL TREATMENT FOR NON-COMPRESSIBLE FLUIDS.

In the case of a non-compressible fluid flowing through tubes whose linear dimensions are proportional to  $r$ , the tubes being geometrically similar, the variables concerned are the pressure gradient  $dp/dx$  at any chosen point in a chosen direction (the pressure differences due to difference of level being deducted) the density  $\rho$ , the viscosity  $\mu$ , and the velocity  $u$ .

Then we have for all geometrically similar systems, by the method of dimensions  $\varphi_1(dp/dx, u, \rho, \mu, r) = 0$ , provided no other variables have to be considered.



Hence, without loss of generality, we may write

$$\varphi_2 \{ (r/u^2 \rho) d\bar{p}/dx, ur\rho/\mu \} = 0$$

or

$$f_1(ur\rho/\mu) = -(r/u^2 \rho) \cdot d\bar{p}/dx \quad \dots \dots \dots (1)$$

where  $f_1$  is a function which changes only if the form of the boundary or the state of turbulence of the entering liquid be changed.

In these equations,  $u$  may be defined as the average velocity at any chosen point in any chosen direction, or it may be defined as the mean velocity of the fluid normal to any chosen transverse section of the tube.

In place of the variable  $d\bar{p}/dx$ , we may equally well use the variable  $dZ/dx$ , where  $dZ$  may be defined as the mean value of the work done on unit mass of the liquid in passing a distance  $dx$  from any chosen point, or otherwise  $dz$  may be defined as the mean value of the work done on unit mass of the liquid in crossing the space between two chosen parallel surfaces a distance  $dx$  apart.

Then, by an exactly similar reasoning, we get

$$f_2(ur\rho/\mu) = (r/u^2) \cdot dZ/dx^* \quad \dots \dots \dots (2)$$

It is easy to show that in the case of a non-compressible fluid flowing through a straight parallel-walled tube,  $f_1 \equiv f_2$ , provided  $dZ/dx$  be defined as the mean work done on unit mass of fluid per unit distance measured parallel to the axis of the tube in the direction of flow.

### III. DIMENSIONAL TREATMENT FOR COMPRESSIBLE FLUIDS.

In the case where the fluid is appreciably compressible, as in the case of a gas, we have two fresh variables to consider, say  $\gamma$  and  $\bar{p}$ , or  $\gamma$  and  $u^2 \rho/\gamma \bar{p}$ . ( $u^2 \rho/\gamma \bar{p} = u^2/c^2$ , where  $c$  is the velocity of sound in the fluid at the point.)

Then by the method of dimensions (provided no other variables have to be considered) we have

$$\varphi_3(d\bar{p}/dx, u, \rho, \mu, r, \gamma, \bar{p}) = 0$$

which may be written without loss of generality

$$\varphi_4 \{ (r/u^2 \rho) d\bar{p}/dx, ur\rho/\mu, \gamma, u^2 \rho/\gamma \bar{p} \} = 0$$

Hence

$$F_1 \{ ur\rho/\mu, \gamma, u^2 \rho/\gamma \bar{p} \} = -(r/u^2 \rho) \cdot d\bar{p}/dx \quad \dots \dots \dots (3)$$

and by similar reasoning

$$F_2 \{ ur\rho/\mu, \gamma, u^2 \rho/\gamma \bar{p} \} = (r/u^2) \cdot dZ/dx \quad \dots \dots \dots (4)$$

In these equations there is again a choice of ways in which to define the variables.

### IV. THERMO-DYNAMICAL THEORY FOR COMPRESSIBLE FLUIDS IN PARALLEL-WALLED TUBES.

It is the aim of subsequent parts of this Paper to show that for flow in a straight parallel-walled tube the function  $F_2$  is almost (if not entirely) independent of  $\gamma$  and  $u^2 \rho/\gamma \bar{p}$ , being only a function of  $ur\rho/\mu$ .

\* For slightly conical tubes this equation takes the form

$$f_2 \{ \} = -(r/u^2 \rho) d\bar{p}/dx + (r/s) ds/dx$$

where  $s$  is the area of the transverse section.



Also, using equation (7), equations (3) and (4) become:—

$$F_1 \{ur\rho/\mu, \gamma, u^2\rho/\gamma p\} = -(r/u^2\rho) dp/dx \quad (10)$$

$$F_2 \{ur\rho/\mu, \gamma, u^2\rho/\gamma p\} = -(r/u^2\rho) \cdot \frac{1-u^2\rho/\gamma p}{1+(\gamma-1)u^2\rho/\gamma p} \cdot dp/dx^* \quad (11)$$

On the second of the above suppositions, we have†

$$\left. \begin{aligned} d(u^2/2) + d(pv) + dU + dZ &= 0 \\ dU + p \cdot dv &= 0 \end{aligned} \right\} \quad (12)$$

Hence

$$dZ/dx = -\frac{1}{\rho} \cdot (1-u^2\rho/\gamma p) \cdot dp/dx \quad (13)$$

and

$$pv^\gamma = \text{constant}.$$

In this case, since the gas expands according to the simple adiabatic law, the density and temperature in the tube (and hence the viscosity) can be directly calculated, and the values of all the variables deduced.

Then, using equation (13), equations (3) and (4) may be re-written (using dashes to denote results of the second supposition):—

$$F'_1 \{ur\rho/\mu, \gamma, u^2\rho/\gamma p\} = -(r/u^2\rho) \cdot dp/dx \quad (14)$$

$$F'_2 \{ur\rho/\mu, \gamma, u^2\rho/\gamma p\} = -(r/u^2\rho)(1-u^2\rho/\gamma p) \cdot dp/dx \quad (15)$$

It will be seen that in equations (10), (11), (14) and (15) the seven variables concerned,  $r$ ,  $p$ ,  $\rho$ ,  $\gamma$ ,  $\mu$ ,  $u$  and  $dp/dx$  occur in the form of four independent non-dimensional products  $ur\rho/\mu$ ,  $(r/u^2\rho) dp/dx$ ,  $\gamma$  and  $u^2\rho/\gamma p$ .

For small velocities all the above four equations (10), (11), (14) and (15) approach that for non-compressible fluids. And for velocities equal to that of sound, equations (11) and (15) would necessitate either the functions  $F_2$ ,  $F'_2$ , becoming zero, or the pressure gradient becoming infinite. The latter is probably the case, since dissipation of energy is always present, and it seems likely that no pressure gradient, however great, would cause a velocity quite as large as that of sound in the gas at the point. This may be supposed one of the causes of Hartshorn's results.‡

The equations (10), (11), (14) and (15) may now be applied to experimental results to find in what way the functions  $F$  depend on  $ur\rho/\mu$ ,  $\gamma$  and  $u^2\rho/\gamma p$ , and to decide if possible which of the hypotheses (6) and (12) is nearer the truth.

## V. DETAILED THEORY USED IN THE EXPERIMENTS.

For measuring the mass  $m$  of air passed per second, a circular orifice in a thin plate was used. If  $R$  be the radius of the orifice,  $p_0$  the small difference in pressure between the fluid on the two sides of the orifice plate, and  $\rho_0$  the density of the fluid, we have

$$m = C \cdot \pi R^2 \cdot \sqrt{2p_0\rho_0} \quad (16)$$

where  $C$  is the coefficient of discharge. The effect of compressibility of the fluid

\* For slightly conical tubes this equation takes the form

$$F_2 \{ \} = -(r/u^2\rho) \cdot \frac{1-u^2\rho/\gamma p}{1+(\gamma-1)u^2\rho/\gamma p} \cdot dp/dx + \frac{(r/s) \cdot ds/dx}{1+(\gamma-1)u^2\rho/\gamma p}.$$

† Eason, *loc. cit.*

‡ Proc. Roy. Soc., 94A, p. 155 (1918).



was arranged to be negligible.\* The effects of viscosity† and situation of the orifice were allowed for by determining the value of  $C$  for water for the same range of values of  $u_0 R \rho_0 / \mu_0$  as occurred in the air experiments. An additional check was obtained by the agreement of all the functions  $f$  and  $F$  for small values of  $u^2 \rho / \gamma p$ .

In all the experiments the fluid entered the narrow tube used through a converging or conical section (Fig. 1). By preliminary experiments it was found that no appreciable error would be introduced by neglecting the friction in the entrance cone. For experiment showed that this small effect could be considered as equivalent to a slight increase in the length of the tube at the entrance end; and the distance between the part experimented on and the effective entrance to the narrow tube does not occur in the calculations.

Considering now the first supposition, that gave equation (8), let  $p_1, \rho_1, v_1, \theta_1$

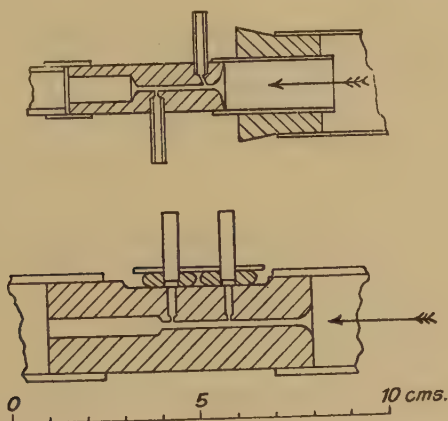


FIG. 1.

refer to the gas before it enters the cone,  $p_2, \rho_2, v_2$  refer to the gas at the narrow end of the cone. Then by the theory for frictionless adiabatic gas flow‡ we have

$$A = (m/s)^2 \cdot \frac{\gamma-1}{\gamma} = 2p_1 \rho_1 \left\{ (p_2/p_1)^{\frac{2}{\gamma}} - (p_2/p_1)^{\frac{\gamma+1}{\gamma}} \right\} \quad (17)$$

Hence 
$$B = A v_2^2 + 2p_2 v_2 = 2p_1 / \rho_1 \quad (18)$$

Also 
$$\theta = \frac{2\theta_1}{1 + \sqrt{1 + AB/p^2}} \quad (19)$$

and 
$$\rho = (1 + \sqrt{1 + AB/p^2}) \cdot p/B$$

where the temperatures are on the absolute scale.

\* J. L. Hodgson, Proc. Inst. Civil Engrs., Vol. 204, Part 2, pp. 42-51 (1916-17).

† Proc. Phys. Soc., Vol. 33, Part 4, p. 225 (1921).

‡ J. L. Hodgson, *loc. cit.*

In the case of the second supposition, equation (13) showed that the expansion would then be simple adiabatic. Hence we have in this case

$$\begin{aligned}\theta &= \theta_1 \cdot (p/p_1)^{(\gamma-1)/\gamma} \\ \rho &= \rho_1 \cdot (p/p_1)^{1/\gamma}\end{aligned}\quad \dots \dots \dots (20)$$

Equations have now been derived enabling corresponding values of the functions and  $u\rho/\mu$ ,  $u^2\rho/\gamma p$ , &c., at points in the tube to be obtained from experimental results.

#### VI. EXPERIMENTAL ARRANGEMENTS.

The experiments were carried out using fine parallel-walled tubes of circular section, having a conical transition at each end to tubing of larger diameter (Fig. 1). The cone through which the fluid entered the narrow tube, was carefully rounded so as to avoid any sudden change of direction at the junction.

The diameter and length of the tubes were chosen so as to enable the air, pumped through by the rotary pump, to attain velocities comparable with that of sound in the air. (Velocities over two-thirds of that of sound were reached.)

The first tube used was drilled in brass, and at three points along it, small side holes were drilled for connection with manometers. It was found that two only of these tubes were necessary, as the pressure differences were sufficiently small to enable a mean value of the pressure gradient (deduced from the pressure difference) to be used. The distance between the side tubes used was 1.33 cm., and the radius of the tube  $r=0.0697$  cm.

The second tube made by drilling a 1 in. diameter rod of "Bakelite" (or ambroid) (of thermal conductivity about  $0.000,6$  cal. cm.<sup>-1</sup> sec.<sup>-1</sup> temp.<sup>-1</sup>) was used in order to investigate the effect of the conductivity of the walls, and also the effect of the distance between the entrance cone and the part of the narrow tube experimented upon. This tube had a radius  $r=0.0770$  cm., and the distance between the side tubes was 1.504 cm. The sizes of the tubes and orifice were measured by means of an Adam Hilger travelling microscope.

#### *Air Experiments.*

The orifice used in the air experiments had a radius  $R=0.2579$  cm., and was drilled in steel sheet 0.015 cm. thick. The orifice plate was fitted between flanges in a tube of 2.5 cm. diameter. An inclined "U" tube paraffin manometer (I Fig. 2) was used to measure the difference in pressure between the two sides of the orifice; and the manometer was previously standardised by means of a simple water manometer read by a travelling microscope. The pressure difference  $p_0$  never exceeded that due to a column of water 5 cm. high, in any of the flow experiments. The value of  $C$  obtained by water tests at "corresponding speeds" was 0.625. This is lower than the corresponding value 0.65 given in the Author's Paper\* owing to the difference in the points between which the pressure difference is measured. The pressures and pressure differences were measured by means of mercury and paraffin manometers. Thermometers were inserted to measure the temperature of the air at the orifice and before it entered the narrow tube.

\* Proc. Phys. Soc., Vol. 33 Part 4, p. 226 (1921).

In the first set of experiments the air passed from the room through the orifice, and experimental tube, and thence through a reservoir (to decrease pulsation) to the pump and was returned into the room.

In the later experiments the air was pumped round a closed system of pipes (Fig. 2), and bubbled through concentrated sulphuric acid during its circulation, in order to prevent any water vapour being present. A large reservoir  $V$  was also inserted to investigate more fully any possibility of error due to pulsations. The orifice  $O$  was in some cases placed on the other side of the experimental tube  $T$ . The velocity of the gas could be changed by altering the current supplied to the motor driving the pump  $P$ ; and  $u\rho/\mu$  and  $u^2\rho/\gamma p = u^2/c^2$  could be altered independently of one another by decreasing the amount of air in the system by a hand vacuum pump.

#### Water Experiments.

In these experiments the entrance conditions were kept as similar as possible to those in the experiments using air. Water passed through the apparatus from

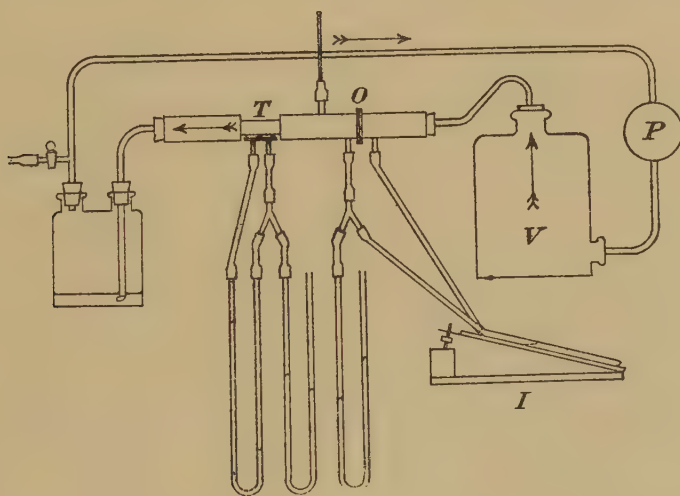


FIG. 2.

the water supply, and then passed into measuring cylinders in order to determine the rate of flow. The temperature of the water was measured in order to determine its viscosity. Care was taken to prevent any air, liberated from the water, entering the manometers or accumulating in any part of the tubes. Little difficulty was experienced on this account except at the largest rates of flow.

#### VII. DISCUSSION OF RESULTS.

From the observed values of the mean pressure gradient and mean pressure in the tube, together with the temperature and pressure of the gas before it enters the tube, and the mass of air passing per second deduced by means of the orifice, all the quantities in equations (10), (11), (14) and (15) can be deduced. The water experiments similarly enable the expressions (1) or (2) (which are identical in the



case of parallel tubes) to be evaluated. If any one of the functions  $F$  is independent of  $\gamma$  and  $u^2 \rho / \gamma p$ , it should yield values equal to those of  $f_2$  for the same value of  $ur \rho / \mu$ . And by simple considerations it is evident that  $F_2$  is most likely to satisfy this condition.

In Fig. 3 values of  $f_1 \equiv f_2$  and  $F_1$  and  $F_2$  are plotted as ordinates against values of  $ur \rho / \mu$ , for the brass tube. Similar values for the Bakelite tube are plotted in Fig. 4. From the earliest experiments only a few points taken at random are inserted. Owing to the entrance conditions and to the nearness of the section experimented on to the entrance, there is no region of instability such as is shown by the dotted curve derived from Stanton's\* experiments on the flow of fluids through brass tubes when the density was sensibly constant. This suppression of the region of instability was noticed in previous experiments.†

It will be seen that though the values of  $f_2$  and  $F_1$  differ considerably, the values of  $f_2$  and  $F_2$  only differ by a small amount even at the largest values of  $u^2 \rho / \gamma p$ , when the velocity of the air exceeded two-thirds of that of sound in the air. Also it was found that both  $f_2$  and  $F_2$  exhibit a similar suppression of the region of instability. If a more powerful pump had been available a longer tube might have been used, to prevent the suppression. But it is not important for the present investigation.

The effect of the adiabatic (or second) supposition of section IV. is shown in the case of extreme points in Figs. 3 and 4, the values of  $F'_1$  and  $F'_2$  being indicated by means of arrows. It will be seen that these points differ more from the values of  $f_2$  than do those of the first supposition.

The value of  $F_1$  for any one value of  $ur \rho / \mu$  was varied by changing  $u^2 \rho / \gamma p$  by pumping some air out of the system by means of a hand pump before the experiment. This illustrates clearly the fact that, not only do  $F_2$  and  $f_2$  agree better than  $F_1$  and  $f_2$ ; but also the values of  $F_2$  are similar for any one value of  $ur \rho / \mu$ , even if  $u^2 \rho / \gamma p$  be changed, whereas this is not so for the values of  $F_1$ .

Before estimating the values of possible errors it may be stated that no difference in the agreement of  $F_2$  and  $f_2$  was found by removing any moisture in the air, by decreasing any chance of pulsations considerably, nor by changing the conductivity of the walls and the distance between the tube entrance and the part experimented upon.

If air about half saturated with water vapour be used, and if all the water vapour were condensed in the experimental tube, it is estimated that in some of the cases the heat liberated might be twice as great as that produced by turbulence. Thus the uncorrected values of  $F_2$  might be too great by about twice the difference between  $F_2$  and  $F'_2$  shown in Figs. 3 and 4. That this does not occur implies that the vapour does not condense appreciably and produce changes in temperature, density and pressure in the tube during the thousandth of a second that it takes to pass along the tube under these conditions.

That change in conductivity of the walls is unimportant is probably due to the fact that in any case conduction is rendered very small by the presence of a thin layer of air moving in non-turbulent flow along the walls of the tube. Thus for typical conditions this layer of air may have a thickness of about 0.003 cm. And

\* Stanton and Pannell, *Phil. Trans., A*, Vol. 214, p. 299.

† *Proc. Phys. Soc.*, Vol. 34, Part 5, p. 195 (1922).

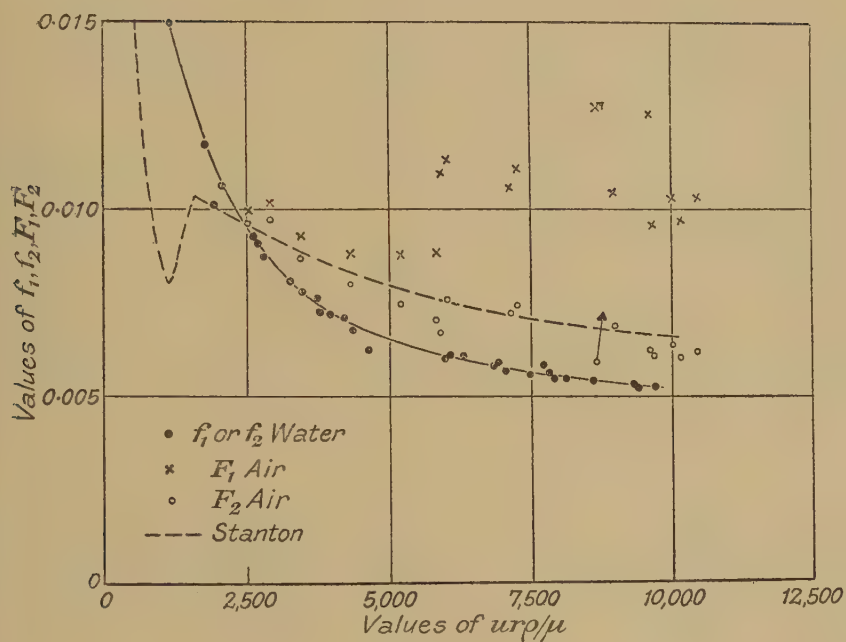


FIG. 3.

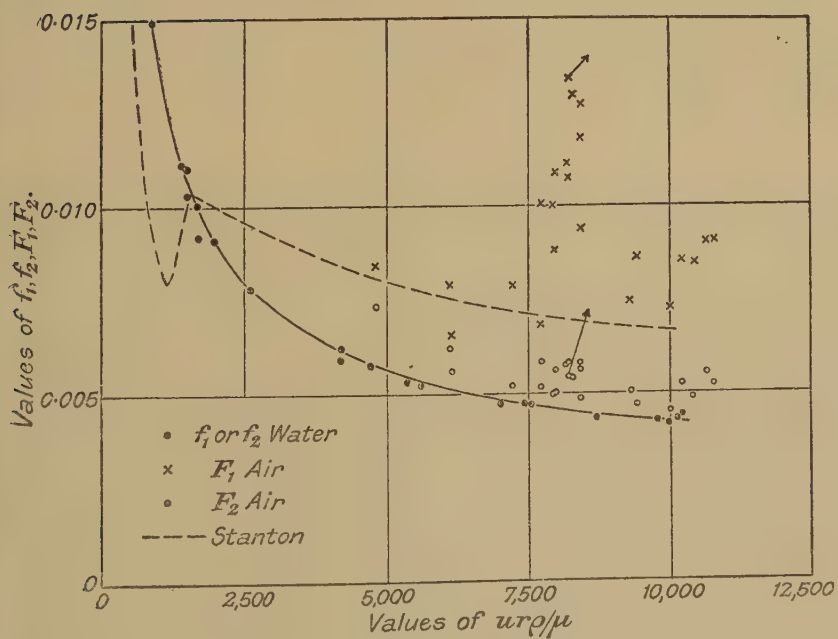


FIG. 4.

in this case the heat conduction (*even if this layer were the only thing that prevented conduction*) would produce less error than the difference between the values of  $F_2$  and  $F'_2$ . By making the walls of a material of low heat conductivity (0.0006), the error was decreased to about a tenth of this value when a steady state was reached. But the error is probably small in any case.

Pulsations were throughout small, and were in the later experiments negligible. Any error due to nearness to the entrance-cone is small, as it is only a second order error on the error next to be considered.

Error due to the effect of the variation in the variables over a transverse section (mentioned in Part IV.) remains to be considered. All the functions  $f$  and  $F$  given by equations (1), (2) and (10), (11), (14) and (15) are in agreement for small values of  $u^2 \rho / \gamma p$ , as is well known. Hence the effect of the variation of the variables over a transverse section is to cause slight errors in the estimates of the mean temperature, density and viscosity in the tube, and to cause the factors

$$\frac{1 - u^2 \rho / \gamma p}{1 + (\gamma - 1) u^2 \rho / \gamma p} \quad \text{and} \quad (1 - u^2 \rho / \gamma p) \quad . . . . . (21)$$

to be in error. The magnitude of this error will probably depend on the values of  $ur \rho / \mu$  and  $u^2 \rho / \gamma p$ , but an estimate of it may be obtained by considering a similar case depending on the variation of  $u^2$  over the transverse section. In this case the neglect of any correction causes the experimental values of  $dp/dx$  to be about 12 per cent. higher than would have been expected by simple calculation.\*

Thus the effective value of  $u^2 \rho / \gamma p$  for gas flowing through the transverse section might be expected to be very approximately 12 per cent. greater than that derived from squaring the mean velocity  $u$ ; and hence the experimental ratio of  $f_2/F_1$  should differ correspondingly more from unity than the ratio  $F_2/F_1$  calculated from the above factors (21), using the mean velocity  $u$ . This effect accounts for the values of  $F_2$  being higher than those of  $f_2$ . This is exhibited more clearly in Fig. 5, in which the full curve represents

$$\frac{F_2}{F_1} = \frac{1 - u^2 \rho / \gamma p}{1 + (\gamma - 1) u^2 \rho / \gamma p}$$

for different values of  $u^2 \rho / \gamma p$ , and the points represent values of  $f_2/F_1$  obtained from the experiments using air and water. The dotted curve is derived from the full curve, by decreasing all the abscissæ in the ratio 100 to 112. Any residual errors may be due to traces of conduction, general experimental errors, or to an underestimate of the error now being considered. It is useless to continue the dotted curve to large values of  $u^2 \rho / \gamma p$ , on account of the fact that it must not cut the axis before the velocity equals that of sound; or otherwise, because the correction depends on the velocity variation over a transverse section, which probably tends to zero for high values of  $u^2 \rho / \gamma p$ . The experiments do not give enough evidence to tell how the factor this error introduces varies with  $u^2 \rho / \gamma p$  and  $ur \rho / \mu$ .

Finally, it is clear that the result of the investigation is to show that when no heat passes through the walls, there is strong evidence for the equation—

$$dp/dx = - \frac{r}{u^2 \rho} \cdot \frac{1 - u^2 \rho / \gamma p}{1 + (\gamma - 1) u^2 \rho / \gamma p} \cdot \Phi(ur \rho / \mu) \quad . . . . . (22)$$

\* Lea's *Hydraulics*, p. 147 (1916); and *Proc. Phys. Soc.*, Vol. 34, Part 5, p. 194 (1922).



derived from equation (11) (in the derivation of which the first supposition (equation (6)) was used), where  $\Phi$  is now supposed independent of  $\gamma$  and  $u^2 \rho / \gamma p$ , and dependent only on  $u r \rho / \mu$ , wall roughness and entrance conditions. The agreement with the first supposition implies that the pressure gradient is affected as if heat were liberated in any length of the tube, the amount liberated being approximately the equivalent of the work done against frictional forces in the same length of tube. The neglect of the variation of  $u^2 \rho / \gamma p$  over any transverse section, however, causes an error at high speeds.

For liquid or slow gas flow this equation reduces to the usual equation—

$$dp/dx = -\frac{\gamma}{u^2 \rho} \cdot \Phi(u r \rho / \mu) \dots \dots \dots (23)$$

corresponding to equations (1) and (2), where  $\Phi$  is identical with the  $\Phi$  of the previous equation.

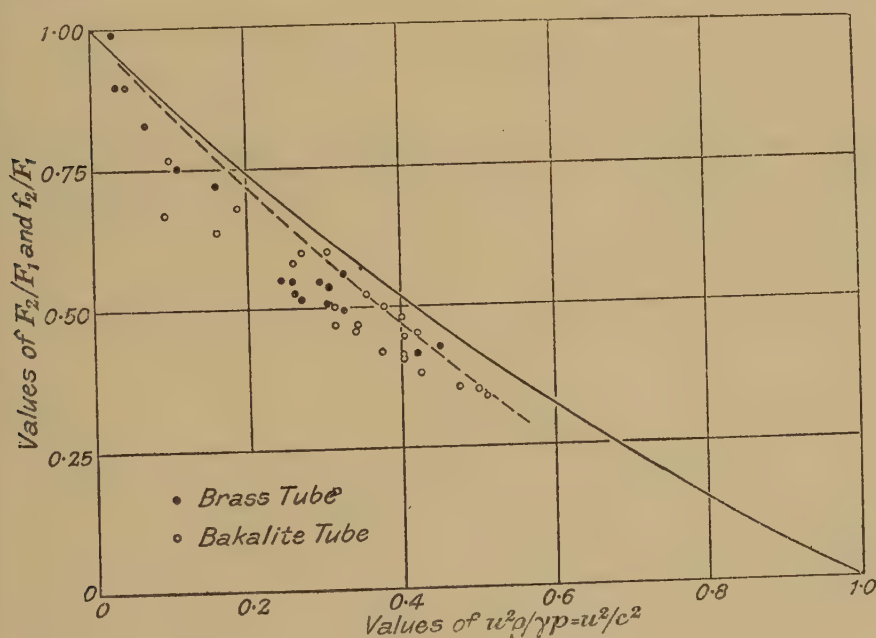


FIG. 5.

Thus, though dynamical similarity cannot be extended to turbulent compressible gas flow throughout the system as a whole, simply by considering corresponding speeds, yet it can be applied to any elementary volume of it, heat conduction being comparatively unimportant, but work being converted into heat by the action of viscosity. The work done against frictional forces is sensibly independent of  $\gamma$  and  $u^2 / c^2$ , and the first hypothesis of equation (6) is supported. A method is thus developed for comparing the pressure-gradients in the tube when any gas is flowing through at speeds comparable with that of sound in the gas, of comparing these cases with those of non-compressible fluid flow, and of predicting one from the other.

The only disagreement found is shown to be probably due to partial neglect of the variation of  $u^2 \rho / \gamma p$  over the transverse section of the tube.

In conclusion, the author would like to thank Dr. W. G. Duffield for the facilities that have enabled these experiments to be carried out, and Mr. J. S. Burgess for help in all experimental matters.

#### DISCUSSION.

Dr. J. S. G. THOMAS asked why such a short experimental tube was used ; results with a length of  $1\frac{1}{2}$  cm. seemed hardly sufficient to support a theory, on account of errors arising from such difficulties as the end effect. He expressed dissatisfaction with the method by which the orifice was calibrated, the value for the orifice coefficient  $C$  (viz., 0.625) appearing of doubtful accuracy. In the arrangement shown in Fig. 2 the orifice-meter is so near the end of the tube that there must be much eddying, and not only will the discharge be liable in such circumstances to pulsate, but, as Lord Rayleigh has fully established, there may be permanent regions of rarefaction and compression in the discharge. Fig. 3 shows no indication of a critical range of velocities, whereas the experimental points are so scattered that some of them might well be expected to fall within that range.

The AUTHOR (in reply) said that the object of using a short tube was to obtain high values of  $u/c$ , the value 0.71 having actually been reached. He did not think the results were much disturbed in consequence, as in the same conditions any irregularities of flow would apply to the water used for calibration no less than to the air. There was certainly no evidence of a critical range of velocities, and such would not be expected near the entrance. He considered that the orifice-constant was not sufficiently in error to affect the final results by more than 2 per cent., as the latter could be compared with the corresponding results for water. He did not think the eddying due to the proximity of the orifice to the tube was serious, in view of the large diameter of the former—namely,  $2\frac{1}{2}$  cm.—and of the shape of the entrance. Any tendency to pulsation was damped out by the provision of two air reservoirs having a capacity of about 11 litres each. There may have been some pressure-striation at the exit end of the tube, but this would scarcely affect the results.

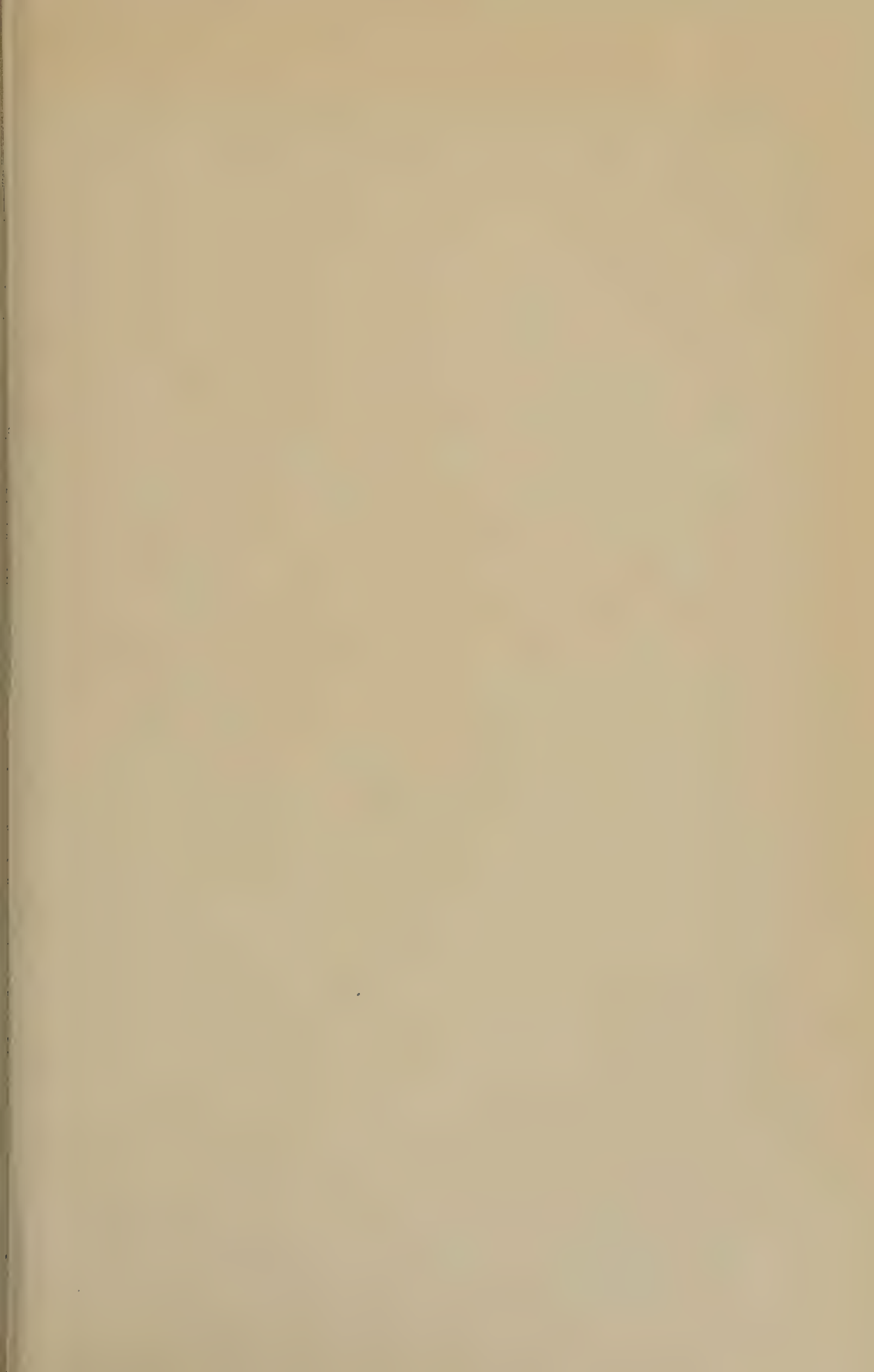






FIG. 1.

[To face page 379.]

## XXVII.—NOTE ON ISRAJ, A REMARKABLE INDIAN STRINGED INSTRUMENT.

By D. B. DEODHAR, *M.Sc.*, Reader in Physics, Lucknow University.

*Received April 28, 1924.*

### ABSTRACT.

1. Israj is a fascinating Indian stringed instrument possessing remarkable acoustic properties. In this instrument all the overtones, which would have been absent according to the Young-Helmholtz law, are brought into prominence by a special form of bridge.

2. The upper portion of the belly is covered by a light skin membrane; and an extra chamois leather ribbon is stretched along a diameter perpendicular to the directions of the vibrating strings. The mechanical impulses of the bridge are carried to the membrane, which is thus energised and emits several harmonics. The chamois leather makes a gradual distribution of pushes from the centre outwards.

3. Thirteen additional tuned wires making an equal tone temperament scale produce a considerable resonance effect.

4. Israj is like a Sitar in that it is a fretted instrument, and it is like a violin in that it is excited by bowing. The tone quality is appreciably richer and fuller than that of a European violin.

### (A) INTRODUCTION.

ISRAJ is a very fascinating combination of Sitar, a Tanpura and a violin, which produces an extremely sweet quality of tone unparalleled in any string or wind instrument. On this instrument melodies are played by bowing the main string, which is generally of steel. (See photograph of Israj in Fig. 1.) Tones of different pitches are elicited by fingering the frets which are adjusted upon the resounding wooden body. There are three additional brass strings stretched on the left side of the main string. Of these strings the one to the extreme left is tuned to a grave C, and the other two are both octaves of this C. The main string is tuned to the major fourth with respect to the adjacent wires. Besides these four wires there are thirteen additional brass strings, stretched upon the sounding board in such a way as to pass freely under the four wires above mentioned. The tension of these thirteen wires is controlled by thirteen pegs, adjusted in a vertical row on a long piece of wood attached to the side of the main sounding board of the instrument. The tension pegs of the main string and its three accompanying strings are situated at the head of the instrument. All thirteen subsidiary wires are tuned in such a way that the wire giving the gravest note C is at the top, and a full equal tone temperament is obtained along the wires controlled by the descending pegs. It is obvious that each string will pick up its appropriate sympathetic vibrations as the player manipulates his instrument by passing from one fret to another in a variety of ways. It is owing to this augmented resonance that the instrument is characterised by a deep well-sustained mellow tone.

### (B) SPECIAL FEATURES OF THE BELLY AND THE BRIDGE.

Unlike the belly of a European violin, the upper portion of the belly of Israj consists of a stretched membrane of light skin; and upon this membrane, along a diameter perpendicular to the length of the main string, is spread a ribbon of chamois leather, the width of the ribbon being usually about a centimetre. A curved

wooden bridge "A" (Fig. 2) is placed in the centre of this ribbon. The two legs of the bridge are approximately of the same type as those of the bridge of a violin; but the top surface, instead of being a sharp edge, has considerable width and slopes

A

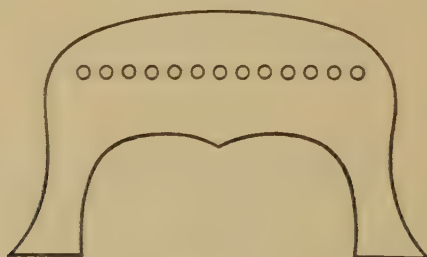


FIG. 2.

regularly both ways in the direction of the strings, as shown in Fig. 3. The strings of a violin usually pass over the bridge at a sharp angle, but in this instrument the four strings pass over the bridge at grazing incidence, and the other set of thirteen



FIG. 3.

strings is arranged so as to pass through the holes made in the wall of the bridge, one hole for each wire (see Fig. 2).

In addition to the wooden bridge A there is another ivory bridge B, 4 cm. long

B

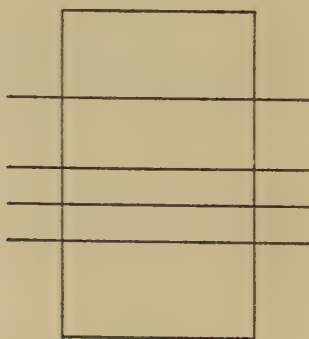


FIG. 4.

and 1 cm. wide, at the top of the instrument, as shown in Figs. 1 and 4. The four wires pass over this bridge in such a way as to lie completely flat on it; and the



height of the bridge is such that a little pressure of the finger is sufficient to make contact at any desired fret.

#### DISCUSSION OF THE MECHANISM OF VIBRATION.

The value of a musical instrument naturally depends on its ability to imitate and accompany all the gradations of vocal sounds. That is to say, graded tone variations, richness, volume of sound (as Wheatstone has put it) and timbre are the chief assets of a perfect instrument. A European violin is said to be perfect in the sense that it facilitates the production of an impression of continuity of transition when the pitch changes from one value to another, and that a few overtones are elicited with the main note excited by the bow, giving much richness to the tone.

Now the richness and quality of a tone directly depend on the number of overtones which are produced along with it ; and therefore to obtain the fullest quality the aim in view must be to obtain the complete assembly of overtones in the bowed string instrument. In connection with the usual European types of violins the well-known principle of Young and Helmholtz applies, and consequently all the harmonics having nodes at the point of excitation, where the bow presses, are absent.\* It is this absence of some of the harmonics which causes the tone quality to lack in beauty and fullness.

But in the case of *Israj* the top and bottom bridges are so related to the strings passing over them that the harmonic having a node at the point of bowing speaks quite distinctly. The Young-Helmholtz law obviously breaks down in this case, and it may be said that it is owing to the breaking of this law that *Israj* obtains its wonderful richness of tone. A similar breakdown of Young's law has been observed by Prof. Raman† in the case of *Tanpura* and *Vina*.

A comparative photographic study of the modes of vibrations of an *Israj* string and a violin string is very interesting ; and the author has observed that the *Israj* vibration curve shows a distinct manner of amplitude change as compared with the violin curve. Very recently Kar‡ has shown how the zero point upon bowed strings is shifted merely by altering the manner of bowing. Care was therefore taken to bow the two instruments under the same conditions ; and the character of the curves was then found to be quite distinct. This characteristic difference of form of the *Israj* vibration seems to be due to the transfer of energy by regular impulses travelling between the vibrating string and the bridges of the instrument. Naturally we have to assume that this transfer is actuated by periodic impulses working between the surface of contact and the string. This view is corroborated by the observations of Prof. Raman on *Tanpura*, and of Dr. Walker§ on the horizontal pendulum of a seismograph.

A still more interesting feature of *Israj* is the stretched membrane and the leather ribbon on which the bottom bridge is placed. The pulsations are given by the bridge to the string above and the membrane below ; the energy given to the membranous structure is far more effective than that given to bellies entirely made of

\* Rayleigh's Sound, Vol. I, p. 187.

† Proceedings of the Indian Association for the Cultivation of Science, Calcutta, Vol. 7, Parts 1 and 2.

‡ Physical Review, Vol. 20, pp. 150-151, second series.

§ Proceedings of the Indian Association, Vol. 7, Parts 1 and 2.

wood. Again the energy supply is made gradual from the centre outwards towards the periphery by means of the ribbon. The pitches of the series of tones due to the forced vibrations started in the membrane under the bridge which is sending pulses at equal intervals can be evaluated. For instance, using equations of membrane disturbance of the form  $\varphi = Z \cos qt$ , where  $\varphi$  is the displacement and  $Z$  the impressed force, and solving the equations containing Bessel functions, the pitches of the various simple tones and the radii of the nodal circles can be determined according to Rayleigh.\* This makes it clear that the richness of tone quality of Israj is very largely due to the grazing incidence of the strings on the bridges and to the vibrations started in the stretched membrane on account of central impulses acting periodically.

\* Rayleigh's Sound, Vol. 1, pp. 322-326.

XXVIII.—THERMIONIC EMISSION FROM SYSTEMS WITH MULTIPLE THRESHOLDS.

By O. W. RICHARDSON, Yarrow Research Professor of the Royal Society, King's College, London.

*Received June 27, 1924.*

ABSTRACT.

In the introductory part of this Paper reasons are given why it is necessary at the present time to face the question of the connection between thermionic emission and the internal electron levels of the emitting substance. The theoretical problem is attacked from three different points of view, namely, (a) classical statistical mechanics, (b) chemical dynamics and (c) the laws of photo-electric action. As a result of this discussion it seems possible to exclude certain alternatives which might otherwise have appeared not improbable and to formulate a set of rules governing the fundamental phenomena which are not in conflict either with the known facts or with any well-recognised relevant principle.

§ I.

IN this Paper I discuss in a preliminary way the theory of the emission of electrons from the surfaces of condensed systems, such as solid metals, when explicit attention is paid to the contingency that the emitted electrons may arise from specific internal configurations characterised by different amounts of energy. In such a case we may anticipate that the material will possess more than one photo-electric threshold. In fact such multiple thresholds are well established in the X-ray region, so that we have to recognise them as a general property of matter. The problem under discussion possesses features of great intrinsic interest, but apart from that there are several reasons why it should be squarely faced at the present state of development of the subject. Some of these I will now enumerate.

In the case of a single effective threshold it is not difficult to interpret the constants in the formula

$$i = \text{saturation current} = AT^2 e^{-\frac{b}{T}} \quad . . . . . (1)$$

in terms of the properties of the electrons in a single internal configuration, and it might be thought that equation (1) might be taken as test of the occurrence of a single effective threshold, though such a position would hardly be justifiable without further consideration. In any event it appears that in the case of the alkali metals the electron emission cannot in general be represented by equation (1) over any considerable range of temperature, the graph of  $\log i - 2 \log T$  (or  $\log i - \frac{1}{2} \log T$ , which behaves similarly to this function) against  $T^{-1}$ , being not rectilinear, but a curve which is convex towards the  $T^{-1}$  axis. This is shown by my old experiments with sodium,\* and quite clearly by the recent measurements of A. F. A. Young† with potassium. It may be that such anomalies are symptoms of multiple thresh-

\* O. W. Richardson, *Phil. Trans., A.*, Vol. 231, p. 539 (1903).

† A. F. A. Young, *Proc. Roy. Soc., A.*, Vol. 104, p. 623 (1923).



holds. On the other hand, they may be caused by complications arising from contamination of the surface. The point is not so very easy to settle by experiment.

It has long been recognised\* that there is a very close connection, amounting practically to identity in some cases, between the photo-electric and thermionic work functions, which correspond to the index  $b$  in equation (1). The double maxima found by Compton and Richardson† in the photo-electric activity-frequency curve for sodium point to the existence of two distinct thresholds. Again, Young (*loc. cit.*) finds very large differences between the work functions calculated from the thermionic and the photo-electric phenomena in the case of potassium. As in the preceding paragraph it is possible that these various complications may be due to an effect of contamination and not to an intrinsic property of the metal.

In a recent Paper‡ I have shown that the results of the measurements of Davisson and Germer§ of the cooling effect and of the temperature variation of electron emission from tungsten require that the energy of the electrons in the metal before emission should be independent of temperature. A combined consideration of the optical properties of metals with the values of their specific heats has for some time been recognised as leading to a similar conclusion. The most reasonable interpretation of these facts is to assume that all the electrons in solids exist in quantized orbits. Such orbits will not in general be identical with Bohr's atomic orbits. They may be more complex and in general it is extremely unlikely that there will be only one type of orbit with an energy level equivalent to a few volts when measured from a point outside the surface. Even if we were restricted to Bohr's atomic orbits we know from the spectroscopic and X-ray evidence that orbits with a few volts lost energy are not uncommon. In any event the inner electron orbits will be present, and the assumption may not be made without justification that they may simply be ignored in thermionic effects.

The theoretical problem may be attacked from several points of view, which I shall now consider in turn.

## § II. THE CLASSICAL THEORY.

On the general view of the phenomena which I am taking it is inevitable that the methods of the classical dynamics will lead to a solution which is not in agreement with reality. Whilst this detracts from the importance of a solution arrived at in this way it does not destroy its importance entirely. In the first place, the results given by this method follow rigorously from the assumed premises provided the adequacy of the laws of dynamics is admitted, and it is not very long since there was a fairly general acceptance of their universal validity. In the second place the precise character of the difference between the results of the classical dynamics and of the quantum theory is a matter of interest. Finally, the solution given by the classical methods will always be some limiting case of the other. Very often this limiting case has properties of the highest importance.

In considering this and similar problems I shall not, as a rule, deal with expressions for the saturation current density but with quantities such as the

\* O. W. Richardson, *Phil. Mag.*, Vol. 23, p. 615 (1912); O. W. Richardson and K. T. Compton, *Phil. Mag.*, Vol. 24, p. 591 (1912).

† K. T. Compton and O. W. Richardson, *Phil. Mag.*, Vol. 26, p. 563 (1913).

‡ O. W. Richardson, *Proc. Roy. Soc., A.*, Vol. 105, p. 399 (1924).

§ Davisson and Germer, *Phys. Rev.*, Vol. 20, p. 300 (1922).

concentration  $\nu_0$  of the electrons in a space surrounded by the emitting substance in equilibrium at temperature  $T$ . The concentrations are all small enough, in so far as the external space is concerned, so that effects arising from the mutual repulsions and attractions of the electrons and ions are negligible and the pressure exerted by the electrons will be given by the equation

$$p = \nu_0 kT. \quad \dots \dots \dots (2)$$

In so far as electron reflection or its effects may be disregarded the saturation current density will be governed by the relation

$$i = \nu_0 e \sqrt{\frac{kT}{2\pi m}} \quad \dots \dots \dots (3)$$

Fortunately it is not necessary to carry out any actual calculations for this case, as I have already given the solution in a form sufficiently general for the purpose.\* Imagine a cavity in the interior of a thermally and electrically insulated mass of the substance which has attained equilibrium at the temperature  $T$ . If the temperature is not too high the effects of the mutual forces of the electrons in the cavity can be disregarded and their potential energies will be constant in this space except for a limited region close to the boundary, where they are attracted by their electrical images in the surface. If the volume of the cavity is considerable this complication may also be left out of account. Let  $\nu_0$  be the equilibrium concentration of the electrons in the parts of the cavity not too near the boundary. As to the interior of the substance, owing to the closeness of the electrons to each other and to the presence of the positively charged particles which are required to make the atoms neutral, it is clear that the potential energy of an electron will vary from point to point. As the material possesses some kind of structure it will be possible to divide it up into a set of regions characterised by the fact that an electron in any given set of such regions possesses the same potential energy. Let the volumes of each set of such regions as may be necessary be denoted by  $v_1, v_2 \dots v_f$ . Let the corresponding negative potential energies be  $V_1, V_2 \dots V_f$ . These are measured from the potential energy of an electron in the cavity which is fixed arbitrarily at zero, a procedure which involves no loss of generality. If  $N$  is the total number of electrons in the whole system and  $\nu_0$  is the volume of the cavity, then the classical dynamics asserts that in the state of thermal equilibrium

$$\nu_0 = N / (v_1 e^{V_1/kT} + v_2 e^{V_2/kT} + \dots + v_f e^{V_f/kT} + v_0) \quad \dots \dots \dots (4)$$

if the number of sets  $f$  is finite. If there are an infinite number of them and they are continuously connected (4) will be replaced by

$$= N \div \int e^{V/kT} dv \quad \dots \dots \dots (5)$$

where  $V$  is the negative potential energy of an electron at any point and the integral is taken throughout the whole volume of the material and the cavity.

The classical dynamics also asserts that the mean kinetic energy of an electron is the same in each set and equal to  $\frac{1}{2}kT$  per degree of freedom.

\* Phil. Mag., Vol. 23, p. 608 (1912). Electron Theory of Matter, 2nd edition, p. 445 (1916).

In considering these results it will be sufficient to keep to (4), more particularly as we are free to make  $f$  as large as we wish. According to (4)  $\nu_0$  appears to be a function of  $v_0$ . As a practical matter this is perhaps not of much importance because on account of the large magnitude of the exponential terms  $v_0$  is negligible compared with the rest of the denominator unless  $T$  is approaching infinity. However, we wish to deal with a  $\nu_0$  which is a definite property of the material, so that we shall define the concentration in which we are interested as the value to which  $\nu_0$  rises as  $v_0 \div \Sigma(v_1 + \dots + v_f)$  approaches zero, and distinguish it as  $\nu_{00}$ . Then

$$\nu_{00} = N / (v_1 e^{V_1/kT} + v_2 e^{V_2/kT} + \dots + v_f e^{V_f/kT}) \dots \dots \dots (6)$$

We now suppose  $V_1, V_2, \dots V_f$  which are all positive, to be arranged in descending order of magnitude; then we may write (6) as

$$\nu_{00} = N e^{-V_1/kT} / (v_1 + v_2 e^{(V_2-V_1)/kT} + v_3 e^{(V_3-V_1)/kT} + \dots + v_f e^{(V_f-V_1)/kT}) \dots \dots (7)$$

$V_2-V_1, V_3-V_1, \dots V_f-V_1$  are all negative so that the denominator in (7) will always lie between  $v_1$  and  $v_1 + v_2 + v_3 + \dots + v_f$ . Now the structure of the material is a repeating pattern in which the unit is a single molecule or a small group of molecules; so that  $v_1, v_2 \dots v_f$  will all be of the same order of magnitude, each arising as the sum of a repeating contribution from each unit. Thus the denominator in (7) will remain of the same order of magnitude whilst  $e^{V_1/kT}$  changes from infinity to unity. From this we conclude that the temperature variation of  $\nu_{00}$  and, in consequence of equation (3), of the thermionic emission will be determined almost entirely by the negative energy of those internal electrons for which this quantity is greatest.

It may be argued that this conclusion has required not only the classical dynamics but also the co-operation with it of the assumption of a finite number  $f$  of specific internal regions, an assumption which is certainly foreign to the classical theory and probably incompatible with it. But if we apply the same considerations to the integrated form (5) which does not suffer from this restriction, we see that  $\nu_{00}$  and the thermionic current will still be dominated, as regards its temperature variation, by the work necessary to remove an electron from those parts of the atom for which this quantity is greatest.

It would, I think, be a fair inference on the classical theory that the index  $b$  in Equation (1) should be equivalent to some thousands of volts corresponding to the work functions for the  $K$  and  $L$  X-ray levels, instead of being a matter of a few volts, as is actually the case. This is another illustration of the kind of contradiction between the results of the classical theories and the facts which we are now becoming very familiar with.

Before leaving the classical theory, there are two matters that may be referred to. We know that it follows from the second law of thermodynamics, either on the classical or any other theory, that

$$\nu_{00} = A \exp. \int \frac{q}{kT^2} dT \dots \dots \dots (8)$$

where  $A$  is a constant independent of  $T$  and  $q$  is the internal latent heat of evaporation of an electron. It follows that  $q$  is related to the  $V$ 's by the equation:—

$$\int \frac{q}{kT^2} dT = \log \frac{N}{A} - \log \Sigma v e^{V/kT} \dots \dots \dots (9)$$



If we consider the application of (6) to a substance with a single threshold  $V_1$ , it reduces to

$$\nu_{00} = \frac{N}{V_1} e^{-V_1/kT} \dots \dots \dots (10)$$

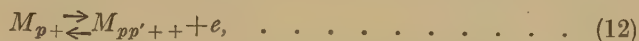
an equation which, as is well known, is not in conflict with the facts of thermionic emission from typical substances.

### § III. THE STANDPOINT OF CHEMICAL DYNAMICS.

I have developed the theory of the emission of electrons from this standpoint in the "Emission of Electricity from Hot Bodies" (1st Edition, p. 49 (1916)). In considering the present problem we can regard the unit of molecular structure of the condensed material, which may not be identical with the chemical molecule of the substance, as ejecting electrons with the production of a series of ionized forms of the molecular unit. If we denote this molecule by the symbol  $M$ , the reactions, in so far as they result in the production of singly ionized forms, may be represented by the set of chemical equations



Each separate level can give rise to a distinct singly ionized form and there will be as many equations in (11) as there are separate levels. If multiple ionization occurs, there will be additional equations. For example, any double ionization can be represented by



and so on. However, in order to simplify the discussion, we shall disregard multiple ionizations for the moment.

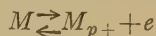
The next step depends upon what is assumed as to the physical constitution of the ionized forms in the condensed phase. An assumption which suggests itself is to treat them as crystalline solids. In that case the concentration of the ionized forms in the condensed phase becomes indefinite and we have to look to the gas phase in order to make any progress towards a solution of the problem. Here we at once meet a difficulty which either is not present or appears to be ignored in the chemical problem. In the chemical problem it is assumed that every species present in the solid phase is theoretically capable of existing in the gas phase and possesses a definite vapour pressure at each temperature. To get ahead with the problem, we shall assume that the same assumption can be made in the case of electron emission. To be more precise, we shall seek to determine the conclusions which follow from the assumption that all the ionized forms which are here assumed to exist as solids in the condensed phase also exist in the gas phase. It must be remembered, however, that this is a very restrictive assumption. It is quite likely that many of the ionized forms in the condensed phase are incapable of existing in the gas phase, and *vice versa*. It follows that whilst the results we shall obtain will certainly apply to some types of ionization by heat, they may not have a very close connection with the central problem of this Paper.

I turn now to the equilibrium in a cavity in the substance at temperature  $T$  of the type considered in the last paragraph. In addition to the electrons, I now consider explicitly the effects of the non-ionized vapour molecules  $M$  and the  $f$

ionized forms  $M_{1+}, M_{2+}, \dots M_{f+}$  required by equations (11). Let  $\nu_{00}$  be the limiting concentration of the electrons in the state of equilibrium in the sense in which it is used in the preceding paragraph. Let  $\nu_1, \nu_2, \dots \nu_f$  be the corresponding quantity for each of the ionized forms and  $\nu$  the concentration of the neutral vapour molecules. For simplicity, I shall suppose that the total pressure is so low that the volume of unit mass of every constituent in the condensed phase is entirely negligible compared with the same quantity in the gas phase. This simplification does not introduce any important restriction, but it involves the necessity for reconsideration of some of the formulæ at sufficiently high temperatures. The consideration of the interaction of the electrons and the ionized forms of equations (11) requires that for any value of  $\nu$  and  $\nu_{00}$

$$\nu_p = \frac{\nu}{\nu_{00}} B_p \exp. \int^T \frac{q_p}{kT^2} dT \dots (p=1, 2, \dots f) \dots (13)$$

Here  $B_p$  is independent of  $T$  and  $q$  is the internal heat of the reaction



There are  $f$  of equations (13).  $\nu$  is evidently definite,\* being determined by

$$\nu = B \exp. \int^T \frac{L}{kT^2} dT \dots (14)$$

where  $B$  is independent of  $T$  and  $L$  is the molecular internal latent heat of the neutral form  $M$ . But for any  $M_{p+}$  which exists in the gas phase and also as a solid or other condensed form in the condensed phase,  $\nu_p$  must also be definite, satisfying

$$\nu_p = C_p \exp. \int^T \frac{L_p}{kT^2} dT, \dots (15)$$

where  $C_p$  is independent of  $T$  and  $L_p$  is the internal latent heat of evaporation of  $M_{p+}$ . Thus

$$\nu_{00} = \frac{BB_p}{C_p} \exp. \int^T \frac{L + q_p - L_p}{kT^2} dT \dots (16)$$

It follows that  $\nu_{00}$  is definite provided there is a single ionized form which exists both in the gas phase and as a condensed form in the condensed phase.

I can, however, show that  $\nu_{00}$  is definite without this restriction. Imagine the cavity to be provided with a cylindrical extension to the external boundary of the substance. The walls of the cylinder are perfectly reflecting both for matter, electrons and radiation, and it is provided with a movable piston whose inner surface possesses the like properties. Across the cylinder and close to the inside surface of the piston place a perfectly reflecting grid whose mesh is so fine that the electrons can get through it, but not the molecules  $M$  nor the ionized forms  $M_{1+}, M_{2+}, \dots M_{f+}$ . Now draw the piston out. The space between it and the grid will be filled with electrons at the concentration  $\nu_{00}$ , but without any of the molecules  $M$  or the ionized forms. By considering the pressure exerted on the piston by these electrons and

\* By definite, I mean completely determined by the temperature and the nature of the substance

making the increment of entropy a complete differential in  $v$  (volume) and  $T$  for a virtual displacement of the piston I find

$$v_{00} = A \exp. \int \frac{q}{kT^2} dT, \dots \dots \dots (17)$$

where  $A$  is independent of  $T$  and  $q$  is the change of energy when an electron is moved from the condensed phase to the space between the piston and the semi-permeable membrane. As the energy of an electron is identical on the two sides of this membrane  $q$  is the internal latent heat of evaporation of an electron as in equation (8).

It follows from (17) and (16) that  $L_p$  the latent heat of evaporation of any ionized form which exists both in the vapour and as a condensed form in the condensed phase must satisfy the equation

$$L_p = L + q_p - q \dots \dots \dots (18)$$

If there is another ionized form  $M_{p'++}$ , which satisfies the same conditions with regard to existence in the two phases as  $M_{p+}$  its latent heat will satisfy the equation

$$L_{p'} = L + q_{p'} - q.$$

So that

$$L_{p'} - L_p = q_{p'} - q_p \dots \dots \dots (19)$$

This result would, I think, be sufficient ground for rejecting the hypothesis that the ionized forms can exist both in the gas phase and as condensed forms in the condensed phase. It would require that the latent heat of evaporation of an elementary crystal singly ionized in the  $K$  level should differ from that of a similar crystal singly ionized in the  $L$  level by the equivalent of the work of transference of an electron from the  $K$  to the  $L$  level. This requirement would be in direct conflict with our knowledge of atomic and crystal structure.

These conclusions are not modified in any essential way if we specifically include the consideration of multiple ionizations. Each new ionized form such as  $M_{pp'++}$  of equation (12) will require for its equilibrium in the gas phase the satisfaction of one independent equation, such as, for example,

$$v_{pp'} = \frac{v_p}{v_{100}} A_{pp'} \exp. \int \frac{q_{pp'}}{kT^2} dT \dots \dots \dots (20)$$

Where  $v_{pp'}$  is the concentration of  $M_{pp'++}$ ,  $v_p$  of  $M_{p+}$ ,  $A_{pp'}$  is independent of  $T$  and  $q_{pp'}$  is the internal heat (per molecule) of the reaction denoted by (12). Each ionized form introduces one independent equation so that there are just enough equations to determine the concentration of each form at any temperature when  $v$  and  $v_{00}$  are given by (14) and (17). We can combine (20) and (13), and, in fact, the result can be generalised in an obvious way to any degree ( $\epsilon$ ) of ionization, giving

$$v_{pp'p'' \dots p\epsilon} = \frac{v}{v_{00}^\epsilon} A_p A_{pp'} \dots A_{pp' \dots p\epsilon} \exp. \int \frac{q_p + q_{pp'} + \dots + q_{pp' \dots p\epsilon}}{kT^2} dT \dots \dots (21)$$

It will be noticed that equations (13), (14), (17), (20) and (21) are independent of any assumptions about the condition or existence of the ionized forms in the condensed phase. They will therefore truly represent the concentration of the various ionized forms in the gas phase. The concentration  $v_{00}$  of the electrons



will not, however, be given in general by (17), but will be determined by the special conditions of the problem. For example, if the condensed phase is not present, and if the gas is neutral as a whole it will be given by the condition

$$\nu_{00} = \sum \epsilon \nu_{pp'} \dots p^e, \dots \dots \dots (22)$$

the summation being extended over all the ionized forms of whatever degree of ionization. It will be noticed from the occurrence of  $\nu_{00}$  or its powers in the denominator of the right hand sides of (13) and (21) that the ionization, and especially the higher degrees thereof, will be depressed by the extraneous introduction of high electron concentrations. This fact may have important applications in spectroscopy. It may be noted that equations (13), (20) and (21) are not restricted to small concentrations.

The results of this section rather suggest that the problem of the levels of the condensed phase might be profitably attacked by treating them as involatile dissolved forms, but, as I have not had time to do this, I shall proceed to consider another method of attack which sheds some light on the problem.

#### § IV. THE PHOTO-ELECTRIC METHOD.

In a series of Papers\* in which I developed the very close connection which exists between thermionic emission and photo-electric action, I showed that it was possible from thermodynamic and statistical considerations to determine the rate of photo-electric activity as a function of the frequency of the exciting light. The form of this function depends on the phenomena which attend the re-absorption of the electrons which return to the emitting substance. If the absorption is complete or if the proportion not absorbed is independent of the temperature (or energy) of the returning electrons the rate of photo-electric activity is of the form

$$\text{Const.} \times (h\nu - \phi) / \nu^3 \dots \dots \dots (23)$$

where  $h$  is Planck's constant,  $\nu$  is the frequency of the active radiation, and  $\phi$  is the energy level of the threshold concerned. The meaning of (23) is that when multiplied by the energy density of the radiation in the stretch  $\nu$  to  $\nu + d\nu$  it gives the rate of ejection of electrons by the photo-electric action of the particular radiation. If, now, we determine the total photo-electric emission of the full radiation characteristic of the temperature  $T$  we know† from thermodynamics that it is identically the same function of  $T$  as the thermionic emission. This statement is a little too broad, but in order to keep the discussion within reasonable bounds I shall suppose we are dealing with a body which satisfies the conditions as to electron reflection which are presumed by (23), in which case it is quite valid. To obtain the total photo-electric emission all we require is to multiply (23) by the relevant energy density, integrate over the effective radiation, i.e., with respect to  $d\nu$  between the limits for  $\nu$  of  $\infty$  and the threshold frequency  $\phi/h$ , and sum over all the levels.

The matter is, however, in one respect not quite so simple, because we have to consider the meaning of the constant in (23) with reference to the different levels.

\* Phys. Rev., Vol. 34, p. 146, p. 393 (1912); Science, Vol. 34, p. 57 (1912); Phil. Mag., Vol. 23, p. 594 (1912); Vol. 24, p. 570 (1912); Vol. 27, p. 476 (1914); Vol. 31, p. 149 (1916); Vol. 47, p. 975 (1924).

† Richardson, Phil. Mag., Vol. 23, p. 615 (1912); Vol. 31, p. 149 (1916).

At this point we can obtain some assistance from the results of the last section, where we saw that we were led into difficulties when we treated the levels in the condensed phase as solids. We therefore adopt the contrary assumption and treat them as the equivalent of gases. That is to say we assume that the activity of the electrons in the different levels, as well as the activity of the different ionized forms, will be proportional to their respective concentrations. Suppose that in a perfectly reflecting enclosure there is a thin layer of the emitting substance, a layer so thin that the emission of electrons is determined by the primary properties of the levels, and is not influenced by the secondary effects arising from molecular encounters and the like. Let each molecule be associated with  $f$  electron levels which give rise on ejection of an electron to the various ionized forms. Let  $n_p$  denote the total number of electrons present in the neutral substance in the (arbitrary)  $p$ th level. Let  $x_p$  denote the number of electrons which have been ejected from this level. Then the rate of ejection of electrons from the  $p$ th level in the presence of unit density of radiation of frequency  $\nu$  to  $\nu + d\nu$  is

$$(n_p - x_p) \frac{A_p c^3}{8\pi} \frac{h\nu - \varphi_p}{k^2 \nu^3} \dots \dots \dots (24)$$

where  $c$  is the velocity of light. The constant  $A_p$  may not be the same for the different levels, but it will not at any rate involve  $n_p$ ,  $x_p$ ,  $\nu$  or  $T$ . The total rate of ejection of electrons from all levels is thus

$$\sum_p \int_{\varphi_p/h}^{\infty} (n_p - x_p) \frac{A_p h}{k^2} \frac{h\nu - \varphi_p}{\frac{h\nu}{e^{kT}} - 1} d\nu \dots \dots \dots (25)$$

To shorten the calculation, I assume that  $e^{\varphi_p/kT}$  is large compared with unity. This introduces no important restriction in the particular problem under consideration, but it reduces (25) to

$$\sum_p (n_p - x_p) \frac{A_p h}{k^2} \int_{\varphi_p/h}^{\infty} (h\nu - \varphi_p) e^{-\frac{h\nu}{kT}} d\nu \dots \dots \dots (26)$$

which, after an integration by parts, becomes

$$\sum_p (n_p - x_p) A_p T^2 e^{-\varphi_p/kT} \dots \dots \dots (27)$$

For a single threshold, and for the usual assumption that the number of ejected electrons is small compared with that of those remaining in the substance ( $x_p$  negligible compared with  $n_p$ ) (27) reduces to

$$A_1 n_1 T^2 e^{-\varphi_1/kT} \dots \dots \dots (28)$$

a form which, since  $A_1 n_1$  and  $\varphi_1$  are constant, is harmonious with the thermodynamic formula (8) or (17), as I have shown elsewhere. For a number of levels and the condition, which will be realised unless the temperature is too high, that every  $x_p$  is  $\ll$  the corresponding  $n_p$  it follows from (27) that the total rate of ejection of electrons is

$$T^2 \{n_1 A_1 e^{-\varphi_1/kT} + n_2 A_2 e^{-\varphi_2/kT} + \dots + n_s A_s e^{-\varphi_s/kT}\} \dots (28.1)$$

For  $T$  small all the terms in the bracket in (28.1) are very small. As  $T$  increases





assumption is similar to one first introduced by Einstein to resolve a well-known difficulty which it would take too long to discuss here. As the external free electrons have the distribution of velocity given by Maxwell's Law, the rate of recombination of electrons with the empty levels may be written\* as

$$8\pi^2\nu_{00}\chi_p\left(\frac{m}{2\pi kT}\right)^{3/2}e^{-\frac{1}{2}\frac{mv^2}{kT}}(F(v)+I_pG(v))v^3dv \quad \dots \quad (31)$$

If we are dealing truly with a unit mechanism, (30) and (31) must balance. They must also agree with the chemical equilibrium formulæ of the type used in §III. In the present notation the formula for chemical equilibrium† is

$$\frac{\nu_{00}\chi_p}{n_p-\chi_p} = T^{5/2}e^{-\varphi_p/kT} \frac{(2\pi m)^{3/2}k^{5/2}\sigma_p}{q_p h^3} \quad \dots \quad (32)$$

where  $q_p$  is the "weight" and  $\sigma_p$  the "symmetry number" appropriate to the level in question. (30) and (31) agree with (32) and (29) and with Planck's radiation formula, provided

$$\psi_p(v) = \frac{8\pi^2 m \sigma_p}{q_p c} F_p(v) (h\nu - \varphi_p) \left(\frac{c}{h\nu}\right)^3 \quad \dots \quad (33)$$

and

$$G_p(v) = \frac{F_p(v)/I_p}{e^{h\nu/kT}-1} \quad \dots \quad (34)$$

The rate of emission of electrons from the  $p$  levels will therefore be

$$\int_{\varphi_p/h}^{\infty} (n_p - \chi_p) \psi_p(v) I_p dv \quad \dots \quad (35)$$

and from all the levels

$$\sum_p \int_{\varphi_p/h}^{\infty} (n_p - \chi_p) \frac{8\pi^2 m \sigma_p}{q_p c} (h\nu - \varphi_p) F_p(\sqrt{h\nu - \varphi_p}) \frac{2c}{h^2} \frac{d\nu}{e^{kT}-1} \quad \dots \quad (36)$$

In general, this integral may be very complicated. There are, however, two important particular cases, namely,

$$(1) \quad (h\nu - \varphi_p) F_p(\sqrt{h\nu - \varphi_p}) = \text{const.} \quad \dots \quad (37)$$

corresponding to the inverse square law of force between the returning electrons and the ionised levels, and

$$(2) \quad F_p(\sqrt{h\nu - \varphi_p}) = \text{const.} \quad \dots \quad (38)$$

corresponding to the assumption that the chance of an electron recombining is independent of its energy. An equivalent assumption to this is that the proportion of electrons reflected is independent of temperature.

\* Milne, loc. cit.

† Milne, loc. cit., p. 219.

Taking Case (1) first, let us put

$$(\hbar\nu - \varphi_p)F_p(\sqrt{\hbar\nu - \varphi_p}) = D_p, \text{ a constant,} \quad (39)$$

in agreement with (37). Then (36) becomes

$$\sum_p (n_p - x_p) \frac{16\pi^2 m \sigma_p}{q_p \hbar^2} D_p \int_{\varphi_p/\hbar}^{\infty} \frac{d\nu}{e^{kT} - 1} \quad (40)$$

$$= \sum_p (n_p - x_p) \frac{16\pi^2 m \sigma_p}{q_p \hbar^2} D_p kT \sum_{n=1}^{\infty} \frac{1}{n} e^{-\frac{n\varphi_p}{kT}} \quad (41)$$

If  $e^{-\varphi_p/kT}$  is small for every  $\varphi_p$ , a condition which will always be fulfilled at a sufficiently low temperature, this reduces to

$$\sum_p (n_p - x_p) \frac{16\pi^2 m \sigma_p}{q_p \hbar^3} D_p kT e^{-\varphi_p/kT} \quad (42)$$

for a single threshold, or, if there is one threshold which has an appreciably lower value of  $\varphi_p$  than any of the others, this reduces further to

$$(n_p - x_p) A_p T e^{-\varphi_p/kT}, \text{ where } A_p = \frac{16\pi^2 m \sigma_p}{q_p \hbar^3} D_p k \quad (43)$$

Now, turning to case (2), we have, putting

$$F_p(\sqrt{\hbar\nu - \varphi_p}) = E_p, \text{ a constant,} \quad (44)$$

in agreement with (38), for the total rate of emission of electrons, from (36),

$$\sum_{p=1}^s (n_p - x_p) \frac{16\pi^2 m \sigma_p}{q_p \hbar^2} E_p \int_{\varphi_p/\hbar}^{\infty} \frac{\hbar\nu - \varphi_p}{e^{kT} - 1} d\nu \quad (45)$$

$$= \sum_p (n_p - x_p) \frac{16\pi^2 m \sigma_p}{q_p \hbar^3} E_p k^2 T^2 \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\frac{n\varphi_p}{kT}} \quad (46)$$

If each  $e^{-\varphi_p/kT}$  is small this reduces to

$$\sum_p (n_p - x_p) \frac{16\pi^2 m \sigma_p}{q_p \hbar^3} E_p k^2 T^2 e^{-\varphi_p/kT} \quad (47)$$

For the case of a single threshold for which  $\varphi_p$  is appreciably less than any of the others this reduces to

$$(n_p - x_p) B_p T^2 e^{-\varphi_p/kT}, \text{ where } B_p = \frac{16\pi^2 m \sigma_p}{q_p \hbar^3} E_p k^2 \quad (48)$$

(47, and (48) have the factor  $T^2$  and agree with (28) and (28.1), which were deduced from assumptions about electron reflection equivalent to (44).

So far there is agreement between the unit mechanism method and the more general method of § IV. So long as equivalent assumptions are made in using both methods each makes the rate of electron emission as given by equations of the type of (43) and (48) the same function of  $T$ .

We shall now consider the total concentration of the external electrons  $n_{00}$  as given by this method, confining our treatment to the case where the number of

ejected electrons is small compared with the number in any level in the condensed phase. This case will always exist at a low enough temperature.

From this and the preceding paragraph we see that the rate of ejection of electrons from any one level is given by an expression of the type

$$(n_p - x_p) K_p e^{-\varphi_p/kT}, \quad \dots \quad (49)$$

where  $K_p$  is some function of  $T$  which does not vary rapidly with  $T$ . The rapid variation of the phenomenon with  $T$  is due to the exponential term in (49). This ejection of electrons is balanced on the unit mechanism hypothesis by the capture of returning free electrons to the same level. This goes on at a rate

$$L_p x_p \nu_{00}, \quad \dots \quad (50)$$

when  $L_p$ , like  $K_p$ , varies little with  $T$ , as an equation of the type of (32) has to be satisfied. We, therefore, have

$$L_p x_p \nu_{00} = K_p (n_p - x_p) e^{-\varphi_p/kT} \quad \dots \quad (51)$$

From this we have

$$x_p = \frac{n_p}{1 + \nu_{00} \frac{L_p}{K_p} e^{\varphi_p/kT}} \quad \dots \quad (52)$$

This determines  $x_p$  if  $\nu_{00}$  is given. There is only one of the equations (52) for each  $x_p$ , so that to determine  $\nu_{00}$  some other condition is required. That  $\nu_{00}$  is definite is shown by the thermodynamic equation (8) or (17), which can be shown to be consistent with the condition that all the electrons in the space come out of the condensed phase. We shall therefore impose this condition in the present treatment of the problem which gives us

$$\nu_{00} v = \sum x_p, \quad \dots \quad (53)$$

where  $v$  is the volume of the space. From (52) and (53)

$$\nu_{00} = \frac{1}{v} \sum \frac{n_p}{1 + \nu_{00} \frac{L_p}{K_p} e^{\varphi_p/kT}} \quad \dots \quad (54)$$

If  $T$  is small enough this reduces to

$$\nu_{00}^2 = \frac{1}{v} \sum \frac{K_p}{L_p} n_p e^{-\varphi_p/kT}, \quad \dots \quad (55)$$

the positive root of which is the value of  $\nu_{00}$  required for the physical problem.

## § VI. COMPARISON OF THE RESULTS OF THE UNIT MECHANISM METHOD WITH EXPERIENCE.

To simplify this discussion I shall keep to one threshold as it introduces no important restriction on the results. It is important to notice that the single threshold is not merely a convenient mathematical simplification by way of abstraction, but it is a condition to which all the formulæ of this Paper which are not based on the classical statistical mechanics reduce if  $T$  is small enough, and one  $\varphi_p$  is appreciably less than all the others. It is therefore a case which we should expect





plate system consisting of an electron emitting cathode and an anode maintained at a given difference of potential, not too large but large enough for saturation, the proportion of electrons absorbed by the anode (i.e., not reflected) will be independent of the distance between the plates. On the results of the unit mechanism hypothesis the proportion should be inversely proportional to the distance of the plates apart, on account of the variation of the number of empty levels due to the variation of the surface density which is proportional to the field strength.

# § VII. AN ALTERNATIVE SUGGESTION.

It appears that all the methods considered lead to some difficulty or another, except the thermodynamic treatment leading to equation (8) and the photo-electric theory leading to equations (28) and (28.1). I therefore propose to retain assumptions which are fundamental to these results and to reject the other special assumptions which have been discussed. In particular I shall reject the unit mechanism hypothesis leading to the recombination formulæ considered in the last two paragraphs. Instead I propose the following set of assumptions which appear to do violence neither to any well accepted principle nor to the known facts:—

- (1) The rate of emission of electrons from the  $p$ th level is

$$=A_p(n_p-x_p)e^{-\varphi_p/kT} \dots \dots \dots (59)$$

In accordance with paragraphs (4) and (5),  $A_p$  is some slowly varying function of  $T$ , and most probably is proportional to  $T^2$ .

- (2) The rate of recombination of electrons with the condensed phase is

$$=B\nu_{00} \dots \dots \dots (60)$$

where  $B$  is some as yet undetermined function of  $T$ . This equation involves abandoning the assumption that the electrons ejected from the  $p$ th level in accordance with (59) are balanced by the direct capture of free electrons by that level. It is equivalent to the assumption that the recombination of the free electrons is a matter in which the whole surface of the condensed phase is involved.

- (3)  $\nu_{00}$  is subject to the condition

$$\nu_{00}v=\sum_p x_p \dots \dots \dots (61)$$

which is consistent with the thermodynamic equation (8).

- (4) The rate of filling up of the unoccupied levels is

$$=F_px_p \dots \dots \dots (62)$$

where  $F_p$  is some slowly varying function of  $T$  which depends on the electronic structure of the condensed phase, but does not explicitly involve  $\nu_{00}$ .

Then, for every  $p$  in the steady state

$$F_px_p=A_p(n_p-x_p)e^{-\varphi_p/kT} \dots \dots \dots (63)$$

So that

$$x_p=n_p\left(1+\frac{F_p}{A_p}e^{\varphi_p/kT}\right) \dots \dots \dots (64)$$

The total rate of emission

$$\left. \begin{aligned} &= \sum_p A_p (n_p - x_p) e^{-\Phi_p/kT} \\ &= \sum_p \frac{F_p n_p}{1 + \frac{F_p}{A_p} e^{\Phi_p/kT}} = B \nu_{00} \end{aligned} \right\} \dots \dots \dots (65)$$

and

$$\nu_{00} = \frac{1}{v} \sum_p x_p = \frac{1}{v} \sum_p \frac{n_p}{1 + \frac{F_p}{A_p} e^{\Phi_p/kT}}, \dots \dots \dots (66)$$

so that

$$B = \sum_p \frac{F_p n_p}{1 + \frac{F_p}{A_p} e^{\Phi_p/kT}} \bigg/ \frac{1}{v} \sum_p \frac{n_p}{1 + \frac{F_p}{A_p} e^{\Phi_p/kT}} \dots \dots \dots (67)$$

For a single effective threshold the rate of emission

$$= \frac{F_1 n_1}{1 + \frac{F_1}{A_1} e^{\Phi_1/kT}} \dots \dots \dots (68)$$

$$= A_1 n_1 e^{-\Phi_1/kT} \text{ for } T \text{ small} \dots \dots \dots (69)$$

and

$$B = F_1 \nu, \dots \dots \dots (70)$$

so that

$$\nu_{00} = \frac{1}{F_1 \nu} \times \text{rate of emission} \dots \dots \dots (71)$$

$$= \frac{A_1 n_1}{F_1 \nu} e^{-\Phi_1/kT} \text{ for } T \text{ small} \dots \dots \dots (72)$$

For (69) to agree with (28)  $A$  should be proportional to  $T^2$ , and for these and (72) to agree with the result of applying the most reliable experimental data as to the variation of  $q$  with  $T$  to (57) it is also necessary that  $F_1$  should be proportional to  $T^{1/2}$ .

Whilst the assumptions made in this section might ultimately not prove to be right, there is, I believe, no known reason which would lead us to suspect that they are wrong.

#### DISCUSSION.

The PRESIDENT said that it was a little difficult to discuss this abstruse Paper because it had not been possible to circulate advance proofs before the meeting. Of the several methods, which the author had discussed, classical dynamics led to results discordant with experience, chemical dynamics gave a little more promise, but the application of quantum principles seemed to offer the best chance of a consistent solution. Fellows would look forward to making a careful study of this valuable contribution when it appeared in the "Proceedings."



## XXIX.—A METHOD OF MEASURING VERY SMALL CAPACITIES.

By L. HARTSHORN, A.R.C.S., B.Sc., D.I.C. (from the National Physical Laboratory).

Received May 18, 1924.

## ABSTRACT.

A bridge arrangement is described suitable for the measurement of very small capacities (of the order of 1 micro-microfarad and less) at telephonic frequencies. Means of securing (1) great sensitivity, (2) fineness of adjustment and (3) elimination of capacities to earth and capacities due to the presence of connecting leads, are indicated, so that it is possible to measure the true inter-capacity between any two conductors, e.g., the electrodes of a thermionic valve. As a test of the method, it is used to measure the capacity between two steel balls of 1 cm. radius and at distances apart of 2.25 to 8 cm.

FOR the calibration of condensers of all the more usual ranges required for general laboratory work the Carey Foster bridge is admirable. At the National Physical Laboratory it is used for the determination of the capacities of condensers ranging from 10 microfarads to something like 50 micro-microfarads, and these small values may be determined with care within about  $\pm 0.1$  micro-microfarad. In radio-frequency circuits it often happens that there are certain very small capacities, a knowledge of which is of considerable importance, e.g., the capacity between the electrodes of a thermionic valve. These capacities may only be of the

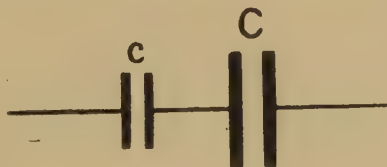


FIG. 1.

order of 1 micro-microfarad or even less, so that they cannot be determined with any certainty on the Carey Foster bridge. The present Paper describes a method which has been used with some success for measuring such small capacities.

In the general testing of variable air condensers, it was noticed that bridges of the Wien type were much more sensitive than the Carey Foster bridge, and for this reason it is often convenient to use such bridges when it is merely desired to compare two variable air condensers. Provided a substitution method is used, and care is taken about the arrangement of the screens of the condensers and the earthed point of the bridge, the Schering modification of the Wien bridge is capable of great precision. Thus in setting out to measure very small capacities, it is natural to turn to a bridge of this type. The difficulty which then arises is the need of a variable

standard of capacity which is capable of extremely fine adjustment, and which has a very open scale already calibrated. For such a standard the writer has used two condensers in series, the capacity of one  $C$  being large compared with that of the other  $c$ . (See Fig. 1.) If  $K$  is the capacity of the combination, then  $K = \frac{cC}{C+c}$ .

Thus

$$dK = \left( \frac{c}{C+c} \right)^2 dC.$$

If now  $\frac{c}{C+c}$  is arranged to be 0.1, then  $\delta K = 0.01 \delta C$ . Thus if  $C$  is changed by 1  $\mu\mu F$  then the capacity of the combination is changed by one-hundredth of this amount. If  $C$  is an ordinary variable air condenser calibrated to read micro-microfarads directly, then obviously by putting a suitable small condenser in series with it, the scale of  $C$  may be made direct reading in hundredths or even thousandths of micro-

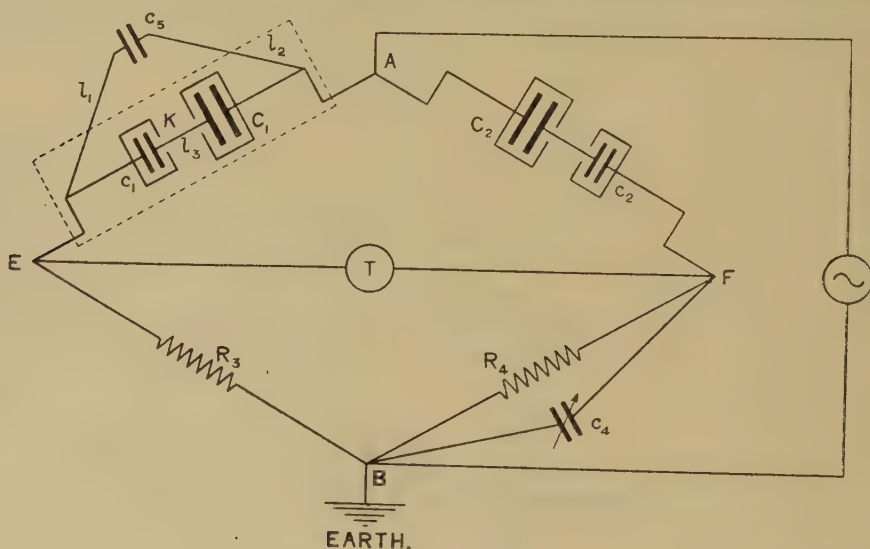


FIG. 2.

microfarads for small changes of capacity of the combination. The change in  $C$  must be so small that the value of  $\frac{c}{C+c}$  is not sensibly altered if the arrangement is to be direct reading. If the change in this ratio cannot be neglected, the change in capacity of the combination must be calculated from

$$\Delta K = \frac{c^2(C_1 - C_2)}{(C_1 + c)(C_2 + c)}$$

where  $C_1$  and  $C_2$  are the two readings of  $C$ , i.e., in calculating the magnification factor  $C^2/(C+c)^2$  the geometric mean of the two values of  $(C+c)$  must be used.

## THE BRIDGE USED.

A bridge found very convenient for this purpose is that due to Schering. The arrangement is shown diagrammatically in Fig. 2. The condenser to be tested is  $C_5$ . By means of a pair of rigid leads  $l$ , it may be inserted in arm No. 1 of the bridge in parallel with the combination standard  $K_1$  made up of the two condensers  $c_1$  and  $C_1$ . Arm No. 2 of the bridge is practically a duplicate of arm No. 1, the idea being to make the bridge as symmetrical as possible. Arms 3 and 4 consist of non-inductive resistances each of 5,000 ohms, and in parallel with one of these is a small variable condenser  $C_4$ , which is for the phase angle adjustment.  $R_3$ ,  $R_4$  and  $C_4$  are all enclosed in metal screens which are connected to the earthed point  $B$  of the bridge. The screens of the large condensers  $C_1$  and  $C_2$  are connected to the point  $A$ . Their capacities to earth are thus merely a shunt across the whole bridge, and do not affect the balance. The screens of  $c_1$  and  $c_2$ , which are much smaller, are connected to the points  $E$  and  $F$  respectively. Their capacities to earth are shunted across the arms  $R_3$  and  $R_4$ . They thus are balanced one against the other. These condensers must be placed as far away from the observer as possible, so that their capacities to earth are as little as possible affected by his movements. These earth capacities can, of course, be entirely eliminated (being located at the corners of the bridge) by Wagner's earthing device, but this is an additional complication, which is not desirable. In the arrangement shown, since the impedances of the arms 1 and 2 are enormous compared with those of 3 and 4, the points  $E$  and  $F$  are not much above earth potential, and thus the earth capacities at these points are only of comparatively small effect.

The extreme fineness of adjustment referred to above would, of course, be of no advantage unless the bridge could be made correspondingly sensitive. It is comparatively easy to do this with the Schering bridge. The voltage applied to the points  $A$  and  $B$  is simply raised until the sensitivity is sufficient. The current carried by the resistances  $R_3$  and  $R_4$  has to pass across the very high impedances in arms 1 and 2 of the bridge, so that this current is always small. The voltage which can be applied is only limited by what the condensers will stand. Several hundreds of volts may be used quite safely in nearly all cases. The bridge takes very little power, and thus a very small transformer is sufficient to step up the voltage to the required amount. In practice the leads to the bridge were connected to a coil of several henries inductance, and this was brought near to the inductance coils of an oscillating valve set until the required voltage was obtained.

The capacities  $c_1$  and  $C_1$  include the capacity between the lead  $l_3$  and the screen of the condensers. This must not be lost sight of in making the calculations. The lead should be of fine wire, and this lead capacity should be included when  $c_1$  and  $C_1$  are calibrated. In any case this lead capacity will be very small, and will not depend very much on the length of lead beyond 10 cm., say. The capacity between the two screens is merely added to the total capacity in arm 1 and does not affect the ratio  $(C+c)/c$ .

## EFFECT OF THE LEADS.

In Fig. 3, let 1 and 2 be the plates of the small condenser, whose capacity is to be determined, and let 3 and 4 represent the leads used to connect the condenser to the bridge. This system of four conductors possesses ten capacity constants—viz.,  $C_{1e}$ ,  $C_{2e}$ ,  $C_{3e}$ ,  $C_{4e}$  the earth capacities, and  $C_{12}$ ,  $C_{13}$ ,  $C_{14}$ ,  $C_{23}$ ,  $C_{24}$ ,  $C_{34}$  the inter-



capacities. The quantity to be determined is  $C_{12}$ . It is to be noted that the bridge arrangement described above measures intercapacities only. The earth capacities on the *A* side of arm 1 may be regarded as a shunt across the whole bridge, having no effect on the balance, while those on the *E* side act as a shunt across  $R_3$ , and are balanced by adjusting  $C_4$ . Thus, in considering the capacity balance we may ignore all capacities to earth. The procedure to be adopted is as follows:—

(a) Connect 1 and 2 to 3 and 4 respectively, and note the bridge reading when balance is obtained. The effective capacity between the leads is now the inter-capacity of the two combined conductors 1, 3 and 2, 4—i.e., of—

$$C_{12} + C_{14} + C_{32} + C_{34}.$$

(b) Break the connection 1-3 without disturbing the lead appreciably, connect 1 to earth, and again take the balance reading. The capacities  $C_{12}$ ,  $C_{14}$  have now

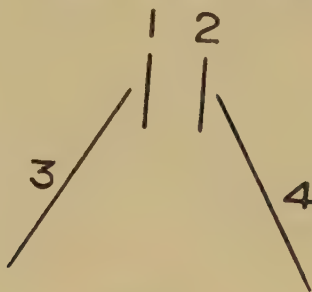


FIG. 3.

become earth capacities, and are thus eliminated, and the effective capacity between the leads is—

$$C_{32} + C_{34}.$$

(c) Again make the connection 1-3, break the connection 2-4 and earth 2. The balance reading now corresponds to a capacity between the leads of—

$$C_{14} + C_{34}.$$

(d) Break both connections 2-3 and 3-4, and earth both 1 and 2. All the capacities except  $C_{34}$  now become earth capacities, and thus the effective capacity between the leads is now  $C_{34}$ .

Taking the differences of the readings (a)–(d), (b)–(d), (c)–(d) we obtain the values of  $C_{12} + C_{14} + C_{23}$ ,  $C_{23}$ , and  $C_{14}$  respectively, and thus  $C_{12}$  may be determined. The positions of the conductors must not be disturbed in taking any of the readings or some of the capacities may be changed. For example, the capacity between the leads  $C_{34}$  is influenced by the presence of the plates 1 and 2, and thus the leads reading must be taken with 1 and 2 in position.

#### EXAMPLE.

As a test of the method it was used to measure the capacity between two steel balls of 1 cm. radius at various distances apart. Each of the balls was supported on a small pillar of paraffin wax. The condensers  $c_1$ ,  $C_1$ ,  $c_2$ ,  $C_2$ , consisted of small fixed mica condensers. Such condensers have the advantage of very small bulk

and thus small capacities between their shields and the leads, and between shields and earth. The values were :—

$$c_1 = 0.000974 \text{ microfarad.}$$

$$C_1 = 0.010116 \quad ,,$$

In parallel with  $C_1$  was a variable air condenser which allowed of the necessary adjustments being made. The mean reading of this condenser was approximately 1,100 micro-microfarads, thus the total value of  $C_1$  was about 0.0112 microfarad, and the magnification factor  $(C_1 + c_1)^2 / c_1^2$  about 160. The leads to the balls were copper wire about No. 30 gauge. Contact was made simply by allowing the ends to touch the polished surface of the balls. The values obtained are given in the table.

TABLE.—Capacity between Steel Balls 1 cm. radius.

Distance between centres.	Capacity in Micro-microfarads.	
	Observed.	Calculated.
2.25 cm.	0.71	0.69
3.0 cm.	0.45	0.43
4.0 cm.	0.32	0.30
6.0 cm.	0.19	0.19
8.0 cm.	0.15	0.14

The capacities  $C_{14}$  and  $C_{23}$  between the balls and the leads varied from 0.18 to 0.12 micro-microfarad depending on the distance apart. The calculated values were obtained by using Russell's\* formula for the mutual capacity coefficients of two spherical conductors. The agreement between the observed and calculated values is as close as can be expected. The observed values are on the whole a trifle higher than the calculated ones. This is no doubt due to the presence of the paraffin wax pillars used to support the balls.

(The value given by Russell for the "capacity between two spheres" of 1 cm. radius and 4 cm. apart is  $0.74 \mu\mu F$ , which is much larger than the above value. This is because the intercapacity between the spheres which is measured by the bridge is not the same as the "capacity between spheres," defined in Russell's Paper. This quantity is a function of the earth capacities of the spheres as well as the intercapacity. The point is fully dealt with in a "Note on the Capacities of Small Air Condensers," shortly to be published in the Journal of Scientific Instruments.)

#### DISCUSSION.

The PRESIDENT referred to the method adopted by the author for securing fineness of adjustment by connecting a large variable condenser in series with a small condenser, whereby variations in the capacity of the large condenser are effectively divided by a substantial factor. The formula for condensers in series being the same as that for resistances in parallel, the method is precisely analogous to the known method in which a small resistance is shunted by a large variable resistance. The analogy may be pressed further, the earth capacities which have to be eliminated corresponding to the resistances of the leads in a resistance bridge.

Mr. T. SMITH called attention to an optical analogy. The Schering bridge used by the author is analogous to an interferometer, the telephone (in which disturbances arriving over

\* A. Russell, Proc. Phys. Soc., Vol. 35, p. 10 (1922).

different paths have to neutralize one another) representing a screen on which interference bands are formed. The electrostatic dimensions of a capacity are those of a length, and the capacity of a condenser may be taken as the analogue of the focal length of a lens, the formula for the capacity of two condensers in series being the same as that for the combined focal length of two thin lenses in contact. Electric oscillations propagated along a wire may be compared to a flat wave, the effect of the condenser corresponding to the introduction of a slight curvature. The analogy fails beyond a certain point owing to the fact that in optics the frequencies are so much higher. In the bridge the sensitivity can be greatly increased by increasing the voltage, in order to measure the smallest capacities; but in the optical case a limit is set by the fact that as the focal length becomes smaller the resolving power of the lens decreases proportionately, though this loss can be compensated by increasing the frequency. In the use of bridge methods for radio-frequency measurements various difficulties arise, and it is worth while to consider whether the optical analogy may not make it possible to employ for their solution the results provided ready-made by optical theory



# XXX.—THE VARIATION OF YOUNG'S MODULUS AT HIGH TEMPERATURES.

By CHAS. H. LEES, *D.Sc., F.R.S.*, JAS. P. ANDREWS, *B.Sc.*, and  
L. S. SHAVE, *B.Sc.*

*Received June 26, 1924.*

## ABSTRACT.

The depression at the free loaded end of a cantilever 4 cm. long in an electric furnace is measured by a microscope. The temperature of the furnace is measured by a platinum resistance thermometer. For aluminium, nickel and platinum, Young's modulus is found to diminish slightly with increase of temperature up to an absolute temperature about half the absolute temperature of the melting point of the material, then to decrease more rapidly, and to tend to zero as the melting point is approached. When the modulus begins to decrease more rapidly viscous flow of the material is first noticed. For fused quartz the modulus increases slowly with rise of temperature up to about 700°C. and then decreases.

## INTRODUCTION.

THIS work was undertaken three years ago\* to supply information as to the variation of the elastic constants of a few typical substances at temperatures approaching their melting points. Up to that time investigators had, with one or two exceptions, restricted themselves to a range of temperature of a couple of hundred degrees or so, as in the case of Miss M. C. Noyes in 1895 and 1896,† and in the 1913 and 1915 Papers of H. L. Dodge.‡ The latter investigated mild steel up to 475°C. in 1915,§ but it was not until 1918 that he investigated the elasticity of Tungsten up to 800°C.|| by a method used by Harrison in 1914¶ upon a rather long Nickel wire. This method depended on the extension of a horizontally stretched wire heated either by an external heating coil or by the passage of a current through the wire. In the latter case the temperature of the wire was estimated by its extension, while in the former case variations of temperature caused great difficulties.

Since the commencement of this research, K. R. Koch and R. Dieterle\*\* have investigated nickel, iron, tin, lead, gold, aluminium, brass and duralumin up to 1,000°C. by a different method. Strips of material 5 to 7 cm. long were suspended vertically in an evacuated furnace heated electrically, and Young's Modulus was deduced from the frequency of transverse oscillations set up in them by resonance. It was found difficult to get them to oscillate at temperatures above 600°C., except in the case of nickel. In all the metals tested Young's Modulus decreased 30 to 50 per cent. on an increase of temperature of 500 to 600°C.

\* Mr. L. S. Shave died on the 26th August, 1922.

† M. C. Noyes, *Phys. Rev.*, 2, 277 (1895). *Phys. Rev.*, 3, 433 (1896).

‡ H. L. Dodge, *Phys. Rev.*, 2, 438 (1913), and 6, 312 (1915).

§ H. L. Dodge, *Phys. Rev.*, 5, 373 (1915).

|| H. L. Dodge, *Phys. Rev.*, 11, 311 (1918).

¶ E. P. Harrison, *Proc. Phys. Soc.*, 27, 8 (1914).

\*\* K. R. Koch and R. Dieterle, *Ann. d. Physik*, 68, 5, p. 441, August 31 (1922).

In the methods so far described the difficulties were increased by the attempt to attain by the same experiment both an accurate numerical value for Young's Modulus and a good comparison of its relative values at different temperatures. In the method we describe below we have reduced the difficulties by aiming at accurate comparative results at different temperatures, the values at air temperatures being already well known.

#### PRINCIPLE OF METHOD.

Since the material under test has to be raised to and maintained at high temperatures, it is desirable to reduce the size of the specimen as far as it is possible to do so without sacrifice of accuracy. This suggests the use of the material in the form of a thin wire. The measurement of the extension of such a wire when of considerable length provides the simplest method of determining the modulus at air temperature. The length might be reduced to a few centimetres and the method of measuring the extension increased in accuracy by the use of a microscope. This method was, however, not considered so promising as that based on the bending of a small beam of the metal fixed at one end, either under its own weight or that of a weight applied at the other end, and this was the method finally adopted.

#### DESCRIPTION OF APPARATUS.

The beam whose deflection was observed consisted of a wire or rod about 5 cm. long, fixed at one end into a screw clamp, and in some cases carrying at the other end a small brass weight, sufficient to give a suitable deflection. The clamp into which it was fixed could be rotated about a horizontal axis perpendicular to the wire in such a way that the wire could be placed at any angle to the horizontal, and the moment of the load about the point of support varied from a maximum when the wire was horizontal to zero when the wire was vertical. The angle of rotation of the support was measured on a scale behind. Alternatively, the wire could be rotated through a convenient angle and the deflection observed in different cases, the moment remaining constant. The wire and support were fitted into an electric furnace 6.7 cm. long, 8 cm. external, and 3 cm. internal diameter. The ends of the furnace when set in the stand were lagged with asbestos, while the clamp was separated from the windings of the furnace by a silica tube. The furnace was run from the 130 volt mains, and required three or four amperes.

The temperature of the enclosure near the clamped end of the wire was determined by a small platinum resistance thermometer inserted through a slot in the flange attached to the clamp. The coil itself was about 1 cm. long, wound of No. 42 pure platinum wire on a slip of mica about 1 mm. wide. This was inserted in a silica tube about 2.3 cm. long, closed at the lower end. The coil was pushed into the end of the tube, and the remainder of the tube plugged with alundum cement, which also served to retain the "dummy" leads, also of platinum wire. The thermometer was standardised, after heating up to 800°C. a few times (to release strains in the wire and to harden the cement) by calibration with ice, steam and boiling sulphur, as usual, with a check point at 800°C., the melting point of NaCl. A Callendar-Griffiths bridge was used to determine the resistance.

The bending of the wire was measured by a microscope provided with a scale in the eyepiece which was focused on the wire near its end through a narrow slit

in an end piece. There was a corresponding slit on the other side of the end-piece through which the light entered the apparatus. The zero position of the wire was determined by rotating the support until the wire was horizontal, first on one side of the clamp and then on the other. As the microscope and clamp are rigidly attached to each other, the mean of the two readings of the eyepiece scale is the position the wire would have taken up if it were entirely unloaded, and its own mass were zero.

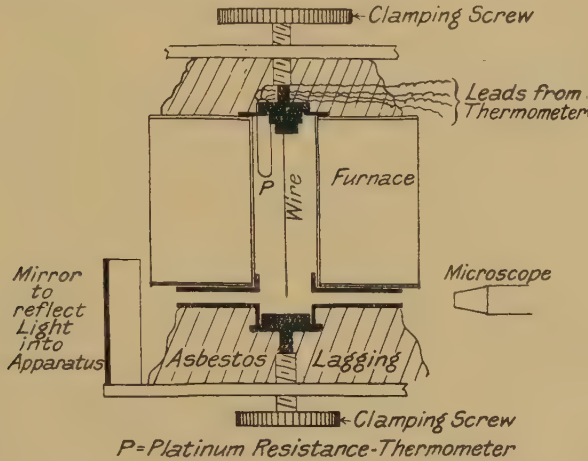


FIG. 1.—SECTIONAL PLAN OF APPARATUS.

### THEORY OF THE METHOD.

The deflections are supposed to be small and in practice were nearly always less than 1 mm. in a direction perpendicular to the length of the wire.

Consider a wire whose axis in the unstrained state is inclined at an angle  $\theta$  to the horizontal.

If  $y$  is the deflection measured perpendicular to the axis, at a distance  $x$  along it when the beam is loaded, and  $I$  is the moment of inertia of the cross-section of the wire, then the bending moment at  $x$  is  $qI \cdot \frac{d^2y}{dx^2}$ , where  $q$  is Young's Modulus. For equilibrium this is equal to the sum of

(a)  $Mg \cos \theta(l-x)$ , due to the mass  $M$  suspended at  $l$ .

(b)  $\frac{A \rho (l-x)^2 g}{2} \cos \theta$ , due to the weight of the wire beyond  $x$ ;  $A$  being the area of cross-section,  $\rho$  the density of the wire.

$$\text{Hence} \quad qI \frac{d^2y}{dx^2} = \left\{ Mg(l-x) + \frac{A \rho g (l-x)^2}{2} \right\} \cos \theta.$$

Which gives on integrating and introducing the conditions that at  $x=0$ ,  $y=0$ , and  $\frac{dy}{dx}=0$ ,

$$y = \frac{Mg}{6qI} (l-x)^3 \cos \theta + \frac{A \rho g}{24qI} (l-x)^4 \cos \theta + \frac{gl^2}{2qI} \cos \theta \left\{ M + \frac{A \rho l}{3} \right\} x - g \frac{l^3 \cos \theta}{6 \cdot qI} \left\{ M + \frac{A \rho l}{4} \right\}.$$



(1) If  $y_1$  is the deflection at the end to which the load is attached,

$$q = \frac{64gl^3 \cos \theta}{3y_1 d^4 \pi} \left\{ M + \frac{3}{8}m \right\} \text{ where } m = \text{mass of wire.}$$

For wires bending under their own weight

$$q = \frac{2\rho gl^4}{y_1 d^2} \cdot \cos \theta.$$

If the method of  $90^\circ$  rotation be used (*see below*)

$$q = \frac{2\rho gl^4}{y_1 d^2}$$

(2) When the thickness of the weight cannot be neglected. Let the thickness be  $2p$ . Then when method of  $90^\circ$  rotation used

$$q = \frac{64gl^2}{y_1 d^4 \pi} \left\{ M \left( \frac{l}{3} + \frac{p}{2} \right) + m \left( l + \frac{p}{6} \right) \right\}$$

In the case of the silica rod, to which this formula applies, the first term was negligible, leaving

$$q = \frac{64gl^2 M}{y_1 d^4 \pi} \left( \frac{l}{3} + \frac{p}{2} \right)$$

#### DETAILS OF MEASUREMENTS.

Two methods were employed.

(1) Method of constant deflections. After tilting the apparatus so that the wire was horizontal, the deflection of the end of the wire at laboratory temperatures was read off in scale divisions of the microscope. At higher temperatures the apparatus was rotated about the horizontal axis perpendicular to the wire just enough to maintain that deflection, the rotation being read on a circular scale behind. This method involved the inconvenience of reading two scales to obtain one value, and very considerable care was required to secure accurate readings. For the silica rod, whose elasticity increases with rise of temperature, the method of constant deflections becomes awkward owing to the difficulty of choosing a suitable deflection before commencing.

(2) Method of  $180^\circ$  rotation. In each case the apparatus was tilted so that the clamped end of the wire was horizontal, the free end extending first to the left, secondly to the right. Half the difference between the readings on the microscope scale gave the deflection when the wire was horizontal, whether the zero were constant or not. This process was performed twice for each observation, and no special difficulty was encountered in obtaining consistent readings. This method was generally used.

*Length of the Wire.*—This was obtained with a travelling microscope to an accuracy of about 0.1 per cent.

*Diameters.*—Were measured by microscope to about 1 per cent. accuracy.

*Magnification of the Observing Microscope.*—The microscope was focused on to a stage micrometer and directly compared. It was found that one small scale division or  $1/10$  of a unit of the scale, was equivalent to 0.00795 cm.

*Temperature Distribution Inside Furnace.*—The temperature of the furnace was raised to about 800°C., and carefully maintained at that temperature for some time, so that steady conditions were obtained. By placing the resistance thermometer at different positions in the furnace, the distribution of temperature was found. The furnace was then allowed to cool, and at one or two temperatures between 800° and that of the room the process was repeated.

It appeared that there was a difference of temperature between the ends of the wire of about 5 to 6 per cent. throughout the range.

It can be shown that the contribution which an element of the wire makes to the final deflection is proportional to the cube of its distance from the free end. Consequently the important temperature is that at the clamped end, and a fairly considerable difference of temperature may be allowed the other end. The temperature measured was in all cases that near to the clamp.

*Procedure.*—After taking an observation at room temperatures, the furnace was heated to about 50°C. or 100°C., and then the heating current reduced. An observation was then taken, preceded and followed by a measurement of temperature. By this means the furnace was employed just as it ceased to rise in temperature, and was about to cool. The two temperature observations generally agreed to within one or two per cent. The mean temperature was recorded.

At the higher temperatures there is always a progressive movement of the wire downwards owing to viscous flow of the material. The observations recorded are those taken immediately after the apparatus was adjusted in position.

#### OBSERVATIONS AND RESULTS.

(1) *Nickel Wire* (1).

Length, 4.770 cm.

Diameter, 0.01038 cm.

No weight on end.

Method of 180 deg. rotation.  $q = 10.53 \times 10^{11} \times \frac{1}{y}$ .

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
15°C.	1.67	$17.6 \times 10^{11}$	
107	1.62	17.1	
217	1.56	16.4	
300	1.54	16.2	
330	1.54	16.2	
352	1.53	16.1	
370	1.48	15.6	
387	1.46	15.4	
410	1.43	15.1	
430	1.38	14.7	Viscous flow first noted.
455	1.34	14.1	
500	1.16	12.2	
600 Viscous	low so marked that readings impossible.		

The elastic curve for this wire shows an upward bend between 300°C. and 400°C. This is in agreement with previous work.\*

\* E. P. Harrison, Proc. Phys. Soc., 27, 8 (1914). K. R. Koch and R. Dieterle, Ann. d. Physik, 68, p. 441 (1922). W. del Regno, Atti. d. Reale Accad. Naz. dei Lincei., p. 105 (1922).

*Note on the Effect of a Magnetic Field.*—The nickel wire is tested in a longitudinal magnetic field due to the heating coil of the oven. It has been found by W. Brown,\* that a longitudinal field of 20 c.g.s. units at about 18°C. produces a diminution of Young's Modulus of only 4 per cent. The effect of the field is not likely to be serious.

## (2) Nickel Wire (2).

Length, 4·800 cm. Diameter, 0·01034 cm. No weight on end.

Method of 180 deg. rotation.  $q = \frac{1}{y} \times (10 \cdot 90 \times 10^{11})$ .

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
18°C.	1·64	$17 \cdot 9 \times 10^{11}$	Viscous flow noticed.
155	1·56	17·0	
314	1·52	16·6	
340	1·53	16·7	
365	1·52	16·6	
380	1·52	16·6	
415	1·48	16·1	
494	1·30	14·2	
570	1·05	11·5	

This wire, taken from the same reel as the first specimen, exhibits precisely the same properties. Viscous flow, as in the former case, begins at about 420°C., and at 600°C. completely masks the elastic effect; the flow appears to begin, and elasticity to decrease rapidly at the same temperature.

## (3) Aluminium Wire (2).

Length, 4·950 cm. Diameter, 0·0125 cm. No weight on end.

Method of 180 deg. rotation.  $q = (2 \cdot 56 \times 10^{11}) \frac{1}{y}$ .

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
12°C.	2·12	$5 \cdot 42 \times 10^{11}$	Viscous flow noticed.
65	1·97	5·03	
100	2·00	5·12	
168	1·73	4·43	
217	1·56	3·98	
252	1·42	3·63	
308	1·10	2·82	
364	1·00	2·56	
418	0·87	2·22	
495	0·66	1·69	

These results are in very close accord with those of Koch and Dieterle,† and with those of H. L. Dodge† up to his highest temperature, 183°C. The value of Young's Modulus at room temperatures appears to be somewhat lower than usual.

\* Proc. Roy. Dub. Soc., April (1917).

† *Loc. cit.*



# Young's Modulus.

411

## Aluminium Wire (2).

Reheated.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
12°C.	2.13	$5.45 \times 10^{11}$	Viscous flow first observed.
60	2.07	5.28	
140	2.02	5.16	
215	1.82	4.65	
290	1.60	4.09	
372	1.30	3.32	
445	0.95	2.43	

## Aluminium Wire (3).

Length, 4.845 cm. Diameter, 0.01192 cm. No weight on end.

Method of 180 deg. rotation.  $q = (2.58 \times 10^{11}) \frac{1}{y}$ .

Temperature.	$\frac{1}{y}$	Young's Modulus	Remarks.
12°C.	2.5	$6.47 \times 10^{11}$	Viscous flow first observed.
75	2.03	5.25	
125	2.22	5.71	
178	1.93	4.98	
223	1.79	4.53	
280	1.54	3.98	
330	1.20	3.11	
382	1.25	3.23	
483	0.98	2.53	

## Aluminium Wire (3).

Reheated.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
10°C.	2.29	$5.92 \times 10^{11}$	Viscous flow first observed.
74	2.29	5.92	
140	2.24	5.79	
192	2.17	5.60	
278	1.87	4.84	
340	1.65	4.76	
440	1.15	2.97	
480	1.11	2.87	

Aluminium wire (3), from the same reel as (2), shows the same characteristics. In both cases the re-heated wire has a greater Young's Modulus. This is in accord with previous knowledge.

Viscous flow appears to set in at about 160°C., although for some wires it may be as low as 125°C. The temperature at which the viscous flow sets in is higher after heat treatment.

If the curve for log (Young's Modulus) against temperature is plotted, the majority of the curve consists of a straight line, indicating that after about 120°C. the law  $\log E = At + B$  is followed closely.

Viscous flow sets in as before at about the same time as this logarithmic law begins to apply.

*Platinum Wire (1).*

Length, 4.665 cm. Diameter, 0.01288 cm. No weight used.

Method of 180 deg. rotation.  $q = (15.15 \times 10^{11}) \frac{1}{y}$ .

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
15°C.	0.876	$13.2 \times 10^{11}$	Viscous flow first observed
86	0.882	13.4	
138	0.875	13.2	
190	0.885	13.4	
236	0.878	13.3	
290	0.874	13.2	
340	0.863	13.1	
390	0.855	12.9	
440	0.854	12.9	
500	0.840	12.7	
565	0.823	12.5	
615	0.809	12.2	
670	0.792	12.0	
752	0.827	12.5	
810	0.834	12.6	
867	0.630	9.54	

*Platinum Wire (1).*

Reheated First Time.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
11°C.	0.878	$13.3 \times 10^{11}$	Viscous flow first observed
65	0.875	13.2	
140	0.910	13.8	
210	0.902	13.6	
280	0.908	13.7	
370	0.909	13.8	
440	0.905	13.7	
546	0.885	13.4	
618	0.851	12.9	
697	0.799	12.1	
805	0.72	10.9	

Reheating again produces an increase in the elasticity, and in the temperature at which viscous flow sets in. This substance is one for which there are no previous

results. It follows the same kind of law as aluminium and nickel over its normal part.

Platinum Wire (1).

Reheated Second Time.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
60°C.	0.823	$12.6 \times 10^{11}$	
160	0.962	14.5	
225	0.936	14.2	
350	0.926	14.0	
468	0.870	13.2	
585	0.861	13.0	
797	0.782	11.8	
820	0.648	9.8	

The wire was merely allowed to cool before repetition. The elasticity is again increased by a second heating, the general shape of the curve remaining the same.

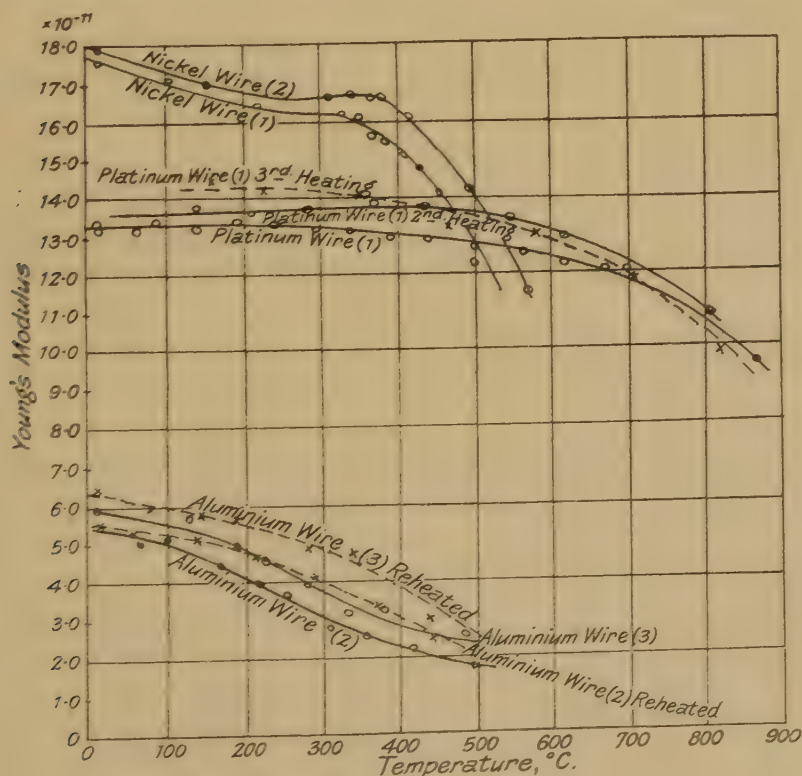


FIG. 2.



*Silica Rod (1).*

Length, 4.870 cm. Diameter, 0.1529 cm. Weight on end, 4.00 gm.

Length of weight, 0.760 cm. Method of 180 deg. rotation.  $q = 6.96 \times 10^{10} \times \frac{1}{y}$ .

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
15°C.	5.72	$3.98 \times 10^{11}$	Viscous flow first observed
100	5.88	4.09	
190	5.66	3.93	
300	6.25	4.33	
382	6.17	4.27	
495	6.07	4.21	
524	6.57	4.56	
545	6.66	4.63	
565	6.66	4.63	
575	7.04	4.90	
586	7.15	4.96	
656	7.70	5.33	
688	6.90	4.79	
740	7.40	5.13	
794	5.0	3.47	
847	5.12	3.57	

The results for silica were obtained with some difficulty, as with the thinnest rod the deflections were extremely small. To hold the rod, a large "blob" was formed on the end by melting in the electric arc, and this was clamped. As the cross section of the rod was not regular, and it had striations on its surface parallel to its length, the absolute values of Young's Modulus are not to be taken as exact, although their comparative values are reliable. The elasticity showed a slight increase with rising temperature, contrary to the usual law. Perrier and de Mandrot appear also to have found an increase.\* Viscous flow observed from 794°C. on.

*Silica Rod (3).*

Length, 4.795 cm. Weight on end, 4.00 gm. Centre of weight, 4.69 cm. from clamp.

Length of weight, 0.760 cm. Diameter, 0.154 cm. Method of 180 deg. rotation.

 $q = (1.19 \times 10^{11}) \frac{1}{y}$ , i.e., magnification of microscope changed.

Temperature.	$\frac{1}{y}$	Young's Modulus.	Remarks.
11°C.	2.90	$3.44 \times 10^{11}$	Viscous flow first observed
70	3.26	3.87	
160	4.00	4.76	
230	4.35	5.18	
340	4.55	5.40	
408	4.86	5.78	
495	4.86	5.78	
590	6.25	7.42	
615	5.70	6.78	
750	5.55	6.61	
805	5.0	5.95	

\* Archives des Sciences, p. 367 (1922).

It was suspected that the weight might be in contact with the sides of the furnace, but, on removal, its position was found to be central. An increase is once more obtained. Viscous flow in this case was observed as early as 615°C.

*Silica Rod (3).*

Reheated without any intervening alteration other than removal from and replacement in the furnace.

$q = 7.865 \times 10^{10} \times \frac{1}{\gamma}$ , i.e., magnification of microscope adjusted to usual value.

Temperature.	$\frac{1}{\gamma}$	Young's Modulus.	Remarks.
20°C.	6.65	$5.22 \times 10^{11}$	Viscous flow first observed.
65	8.35	6.56	
138	9.68	7.12	
215	9.08	7.12	
295	9.60	7.52	
375	10.3	8.09	
480	10.5	8.25	
560	10.9	8.56	
657	11.8	9.28	
726	11.5	9.03	
850	8.35	6.56	

*Silica Rod (3).*

Reheated second time (rod removed from the clamp, rotated through 90 deg. about its own axis, and replaced as before).

Temperature.	$\frac{1}{\gamma}$	Young's Modulus.	Remarks.
30°C.	3.85	$3.02 \times 10^{11}$	
120	4.88	3.83	
225	5.25	4.12	
320	5.88	4.62	
422	4.65	3.66	
530	3.70	2.91	
634	5.88	4.62	
800	6.68	5.23	
880	5.13	4.03	

The rod is evidently not of uniform cross-section.

As the elasticity appears in all cases to increase as the temperature rises, it seemed desirable to make sure of this; the test was therefore again repeated.

*Silica Rod (3).*

Reheated again, after removal from the furnace.

Temperature.	$\frac{1}{\gamma}$	Young's Modulus.	Remarks.
16°C.	5.0	$3.93 \times 10^{11}$	Viscous flow first observed.
138	6.1	4.79	
235	6.25	4.90	
335	6.7	5.26	
442	7.4	5.82	
557	7.4	5.82	
683	8.7	6.82	
805	9.1	7.14	

To make certain that the increase was not a mistake due to any touching inside, a thread of silica was drawn out. Its irregular section did not allow Young's Modulus to be calculated, but it shows the same general increase.

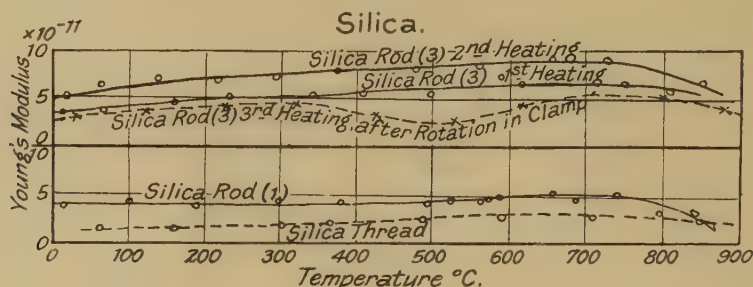


FIG. 3.

*Silica Thread.*

Temperature.	$\frac{1}{y}$	Remarks.
62°C.	1.39	
160	1.56	
302	2.0	
370	2.27	
490	2.7	
590	2.94	
768	2.94	
850	2.7	Viscous flow observed.

The thread was not clamped, but fixed in Alundum cement and baked.

## GENERAL CONCLUSIONS.

1. Young's Modulus for fused quartz changes only slightly over the range of temperature 0°-800°C., for which the expansion is known to be small.\*

2. Young's Modulus for platinum, nickel and aluminium decreases with rise of temperature, at low temperatures slowly, at high temperatures more rapidly. There is a tendency for the change to occur at an absolute temperature about half that of the melting point of the metal. Koch and Dieterle's curves show the same tendency. At about this temperature viscous flow is also first observed. Over the range 0°-600°C. the fractional increase in length of aluminium is 0.016, of platinum 0.0058, and of nickel 0.011.

3. The decrease of the modulus appears to be large where the expansion is large, but the evidence is not yet sufficient to show what part the variability of expansion of the crystalline components of the metals tested plays in the decrease.

\* The fractional increase of length for this rise of temperature is 0.000450. Wheeler, Trans. Roy. Soc. Canada, 8, p. 139 (1914).

## XXXI.—FURTHER EXPERIMENTS ON THE ARTIFICIAL DISINTEGRATION OF ELEMENTS.

By Sir E. RUTHERFORD, *F.R.S.*, Cavendish Professor of Experimental Physics, and  
Dr. J. CHADWICK, Fellow of Gonville and Caius College, Cambridge.

*Received July 19, 1924.*

## ABSTRACT.

In previous Papers the authors have shown that protons can be ejected from the nuclei of B, N, F, Na, Al and P by bombardment with  $\alpha$ -particles, but no certain conclusions could be drawn in cases where the ejected particles had ranges less than 30 cm. in air. In the present experiments these particles were observed at right angles to the path of the incident  $\alpha$ -particles, and the limit of the trustworthy range was thus reduced to 7 cm., or less in some cases. Disintegration was found in the case of the light elements Ne, Mg, Si, S, Cl, A and K, but not in the case of H, He, Li, C and O. The results with Be were doubtful. The following heavier elements failed to show the effect: Ni, Cu, Zn, Se, Kr, Mo, Pd, Ag, Sn, Xe, Au, U. Elements from Ca to Fe have not at present given conclusive results.

The comparative ranges of the ejected particles suggest that the nuclei of the even-numbered light elements are stable while those of the odd-numbered light elements are comparatively unstable. An estimate of the field of force within the atom is deduced from these ranges.

## § I.

IN previous Papers\* we have shown that hydrogen nuclei are ejected from the elements boron, nitrogen, fluorine, sodium, aluminium and phosphorus by bombardment with  $\alpha$ -particles. In these experiments the material subjected to the bombardment was placed immediately in front of the source of  $\alpha$ -particles and observations of the ejected particles were made on a zinc sulphide screen placed in a direct line a few centimetres away, using radium C as a source of  $\alpha$ -rays. The ranges of the  $H$ -particles were in all the above cases greater than the range (30 cm. in air) of free  $H$ -nuclei set in motion by  $\alpha$ -particles, so that, by inserting absorbing screens of 30 cm. air equivalent in front of the zinc sulphide screen, the results were made independent of the presence of hydrogen as an impurity in the bombarded material.

Some of the other light elements were examined at absorptions less than this, but in general the number of particles due to hydrogen contamination of the materials was so large that no confidence could be placed in the results.

At absorptions less than about 12 cm. of air a further complication arises on account of the long-range particles emitted by radium C, in addition to the  $\alpha$ -particles of 7 cm. range. The number of these particles is large compared with the disintegration effect we usually observe, and they therefore mask the presence of disintegration particles within this range.

To overcome these difficulties we have devised a simple method by which we can observe with certainty the disintegration of an element when the ejected particles have a range of only 7 cm. in air. This method is based on the assumption that the

\* Rutherford and Chadwick, *Phil. Mag.*, 42, p. 809 (1921); 44, p. 417 (1922).



particles of disintegration are emitted in all directions relative to the incident  $\alpha$ -rays. A powerful beam of  $\alpha$ -rays falls on the material to be examined and the liberated particles are observed at an angle of 90 deg. to the direction of the incident  $\alpha$ -particles. The slide carrying the source and the material to be bombarded is placed in a brass box, which is evacuated. The ZnS screen is placed outside the box, opposite a hole covered with a sheet of mica of 7 cm. air equivalent.

This method has many advantages. We can now detect particles of range more than 7 cm. with the same certainty as particles of range above 30 cm. in our previous experiments, for the presence of hydrogen in the bombarded material has no effect. This can be shown at once by bombarding a screen of paraffin wax, when no particles are observed on the zinc sulphide screen. On account of the very great reduction in number of  $H$ -nuclei or  $\alpha$ -particles by scattering through 90 deg., the results are quite independent of  $H$ -nuclei from the source or of the long range particles found by Bates and Rogers.\* The latter are barely detectable even when a heavy element like gold is used as scattering material.

A slight modification of the arrangement enables us to examine gases as well as solids.

Since our previous experiments the counting microscope has been considerably improved by a reconstruction of the eyepiece. With the same objective of 0.45 numerical aperture, and with a screen of suitable curvature, we are now able to obtain good definition over an area of 50 sq. mm.

With this experimental arrangement we have found that, in addition to boron, nitrogen, fluorine, sodium, aluminium and phosphorus, which give  $H$ -particles of maximum range in the forward direction between 40 and 90 cm., the following elements give particles of range greater than 7 cm. : neon, magnesium, silicon, sulphur, chlorine, argon and potassium. The numbers of the particles liberated from these elements are small compared with the number from aluminium under the same conditions, varying between  $1/3$  and  $1/20$  of the latter. The maximum ranges of the particles have not been determined with accuracy. Neon appears to give the shortest range, about 16 cm., under our conditions, the ranges of the particles from the other elements lying between 18 and 30 cm. Beryllium, in the form of a flake of the metal, gave a small effect, about  $1/30$  of that of aluminium, but we are not yet certain that the effect may not be due to the presence of a small inclusion of fluoride. The other light elements, hydrogen, helium, lithium, carbon and oxygen give no detectable effect beyond 7 cm.

We have made a preliminary examination of the elements from calcium to iron, but with no definite results, owing to the difficulty of obtaining these elements free from any of the "active" elements, in particular nitrogen. For example, while a piece of electrolytic iron gave no particles of range greater than 7 cm., a piece of Swedish iron gave a distinct effect which was undoubtedly due to the presence of nitrogen, for after prolonged heating in vacuo the greater part disappeared. Similar results were experienced with the other elements in this region.

We have observed no effects from the following elements : nickel, copper, zinc, selenium, krypton, molybdenum, palladium, silver, tin, xenon, gold and uranium. The krypton and xenon were kindly lent to us by Dr. Aston.

It is of interest to compare the above results with those of Kirsch and Pettersson,

\* Bates and Rogers, Proc. Roy. Soc., A., Vol. 105, p. 97 (1924).

communicated to *Nature*, September 15 and November 10, 1923, and described in detail in the *Phil. Mag.*, March, 1924. Kirsch and Pettersson investigated the disintegration of some light elements by our previous method, taking special precautions to avoid hydrogen contamination both in the source and in the bombarded materials. They found that beryllium, magnesium and silicon gave very large disintegration effects, three or four times greater even than that of aluminium, while sulphur and chlorine gave little or no effect. The particles from beryllium had a range of about 18 cm., those from magnesium and silicon about 12 cm.

These results cannot be reconciled with ours, and the probable explanation, in view of the number of particles and their range, is that the particles they observed were the long-range  $\alpha$ -particles emitted by the source.

## § II.

By a small modification of the method described above we have examined some of the lighter elements for disintegration particles of ranges less than 7 cm. of air. It is evident from simple mechanical principles that the range of an  $\alpha$ -particle scattered through a large angle by a light atom is much less than the incident range. For example, an  $\alpha$ -particle of radium C of incident range 7 cm. will, after scattering through an angle of 90 deg. by lithium, have a range of only 1 cm. For carbon the corresponding range is 2.5 cm., for aluminium 4.3 cm., while in the case of a heavy atom like gold the range is only slightly less than the incident range. It is clear, therefore, that we can push the examination for disintegration particles from the lighter elements to smaller ranges than 7 cm. without the complication of the presence of the scattered  $\alpha$ -particles.

Certain difficulties arise, however, which were absent in our previous experiments. In the first place, it is very difficult to avoid contamination due to slow volatilisation or removal of the active matter of the source. In order to get rid of this disturbing effect we found it necessary to cover the source with a thin film of collodion of stopping power about 3 mm. Secondly, any impurity of high atomic weight in the bombarded material gives rise to scattered  $\alpha$ -particles of nearly the incident range.

In these experiments the hole in the brass box containing the source and material to be bombarded was covered with a mica sheet of 2.6 cm. stopping power. This was sufficient to stop the  $\alpha$ -particles scattered through 90 deg. by lithium, beryllium, or carbon. The box was lined with sheets of graphite to avoid any extraneous effect due to scattering from the walls of the box. The arrangement of the bombarded screens was such that no particles could reach the zinc-sulphide screen except by scattering through an angle of at least 90 deg. The box was evacuated during the experiments.

Carbon was examined in the form of a sheet of Acheson graphite. Under these conditions only very few scintillations were observed on the ZnS screen, certainly less than one-tenth of the number observed when aluminium foil as radiator was placed over the carbon. In the case of aluminium the absorption in the path of the disintegration particles was increased to 5 cm. to stop the scattered  $\alpha$ -particles. The effect observed with carbon is so small that it might well be due to a trace of impurity, such as adsorbed nitrogen. There is, of course, still the possibility that particles of range less than 2.6 cm. are liberated from carbon.

This result is in complete disagreement with the experiments of Kirsch and Pettersson (*Nature*, April 26, 1924), who found a very large number of *H*-particles of 6 cm. range.

Lithium was examined in the form of metal. No certain effect could be observed.

The flake of metallic beryllium used in our previous experiments gave a small number of scintillations, the magnitude of the effect agreeing with that recorded above in the investigation for particles of range greater than 7 cm. The effect is so small that it is difficult to be certain that the particles are due to beryllium and not to some occluded fluoride, from which it was prepared.

Nitrogen was tested in the form of paracyanogen,  $C_xN_x$ . A large number of bright scintillations of the  $\alpha$ -ray type was observed. Nitrogen in the form of gas, however, gave few, if any, bright scintillations. As the paracyanogen was prepared from mercury cyanide, it seems probable that the effect observed was due to the presence of mercury.

A similar effect was observed with a sample of powdered boron. These experiments will be repeated with carefully purified materials.

### § III.

The experiments described above, together with our previous experiments, show that all the elements from boron to potassium are disintegrated by bombardment with  $\alpha$ -particles, with the two exceptions of carbon and oxygen. Of the lighter elements, hydrogen, helium, and lithium show no evidence of disintegration, while beryllium gives a small effect which may possibly be due to impurity. The particles liberated in the disintegration appear to be all of the hydrogen type. As yet no evidence has been obtained of the ejection of particles of short range. This is, in a sense, very disappointing, for we had hoped in this way to get direct information about the constituent parts of the nuclei of the lighter elements. We must conclude that these nuclei are either extremely small or of very stable structure, unless it be supposed that such particles have a very short range or are not liberated in the backward direction. It is possible that bombardment by  $\alpha$ -particles of still greater energy may be more successful, and such experiments will be carried out shortly.

One point of general interest appears from the results so far obtained, the marked difference between the behaviour of elements of odd atomic number and of those of even atomic number. The disintegration particles from the odd-numbered elements are, in general, greater in number and of longer range than those from the even-numbered elements. This difference is illustrated in the diagram where the maximum ranges of the particles from the elements from boron to potassium are given by the heights of the rectangles.

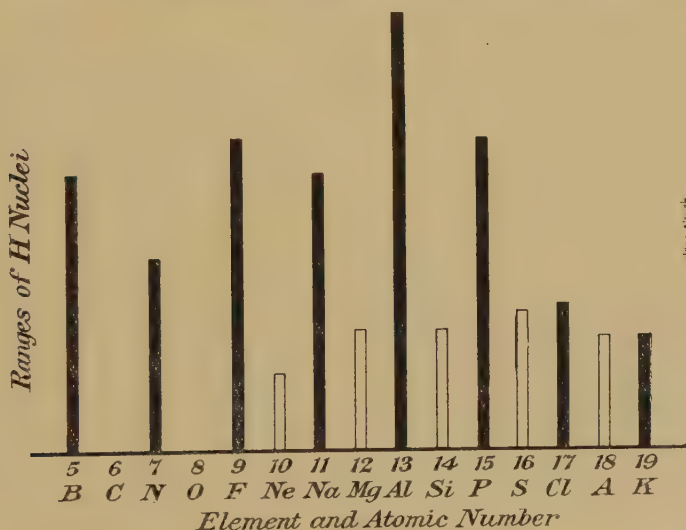
It is difficult to compare the number of particles from one element with that from another as the conditions of experiment cannot be kept constant. For example, one element can be obtained in the form of foil, another only as a powder, another must be used as a gas.

The difference seems to indicate that the nuclei of even atomic number are more firmly built than those of odd atomic number. It is more marked in the lighter than in the heavier elements, as one might perhaps anticipate.



This striking difference between even and odd elements can be paralleled by results obtained in other fields of work. For example, Harkins has shown that even-numbered elements are on the average much more abundant in the earth's crust than odd-numbered elements. Aston has found that in many cases odd-numbered elements have only two isotopes differing in mass by two units, while even numbered elements may be composed of a large number with a considerable range of mass. These differences between the properties of even and odd elements must be of fundamental significance, but with our present knowledge, or rather lack of knowledge, of the detailed structure of nuclei we can only speculate as to the underlying causes.

In our first Paper on the disintegration of elements by  $\alpha$ -particles we gave a picture of the nucleus of a disintegrable atom which consisted of a main central nucleus with one or more hydrogen nuclei as satellites. We assumed that the law of force in this region of the nucleus was one of attraction between like charges.



The force at large distances must become the usual repulsive force varying inversely as the square of the distance. On this view there must, then, be a critical surface around the nucleus at which the force is zero or the potential a maximum. The  $H$ -satellite is, of course, inside this surface and cannot escape from the nucleus with less final energy than corresponds to the potential at this critical surface. Thus the disintegration particles must have a certain minimum range corresponding to this potential. We have examined this point in the case of the elements aluminium and sulphur, using thin films for the purpose. In each case we have found definite evidence of the existence of a minimum range, in sulphur of about 15 or 16 cm., and for aluminium about 13 to 14 cm.

From these results it is possible to form a rough estimate of the potential of the electric field at the critical surface and this comes out to be about 3 million volts for aluminium. The value for sulphur is somewhat greater. This brings out in a striking way the extraordinary minuteness of the nuclei of these elements,



How does that re-emission of light take place? We generally imagine the atoms as containing something of the nature of small radiators, which are capable of forced vibrations under the influence of the primary beam. According to the classical electrodynamics, these radiate waves of the same frequency as the primary; but we must recognise that our present conception of the atom does not very clearly indicate what these radiators may be. Before the advent of the quantum hypothesis we believed we knew what the mechanism was, but the whole question at present seems more mysterious than ever.

#### FLUORESCENT LIGHT.

There is, however, one class of phenomena in which the substance illuminated is capable of radiating light of a different kind from the incident light, and of showing colours which are not present in the illuminating beam. Such substances are said to be fluorescent; they emit a radiation which is, in almost all cases, placed on the long-wave side of the primary radiation; in other words, shifted towards the red end of the visible spectrum. We shall see how these phenomena may be interpreted with the aid of our present conceptions of atomic models.

#### CORPUSCULAR RAYS.

But in addition to these two general types of secondary radiations, waves of ordinary light, ultra-violet light and Röntgen rays excite another secondary radiation of a totally different kind. I refer to the photo-electric effect, which consists in the expulsion, from the atoms composing the illuminated substance, of negative electrons identical with those negative corpuscles which are liberated from the incandescent filament of a wireless three-electrode valve.

These electrons are emitted with a characteristic energy and speed which are definitely related, according to the quantum law, to the frequency of the incident beam: the higher the frequency the faster the electrons. In the case of ordinary light, which is to be considered as having a rather low frequency, in spite of its number of vibrations exceeding  $10^{14}$  per second, the energy of the photo-electrons is small. It only amounts to the energy which could be given to the corpuscle when it is accelerated by a potential difference of a few volts, so that this corpuscular radiation is not important in the case of ordinary light and remained quite unknown up to a comparatively recent date.

*Range of Electromagnetic Rays.*

Rays.	Frequency $\nu$ (sec. <sup>-1</sup> ).	Wave-length $\lambda$ (cm.).	Quantum volts $V$ .	Speed $v$ of photo-electrons (cm./sec.).
Electric ... ..	$10^6$	$3 \times 10^4$	$4.1 \times 10^{-9}$	$3.8 \times 10^3$
Infra-red ... ..	$10^{13}$	$3 \times 10^{-3} = 30\mu$	$4.1 \times 10^{-2}$	$1.2 \times 10^7$
Yellow light ... ..	$5 \times 10^{14}$	$6 \times 10^{-6} = 0.6\mu$	2.06	$8.5 \times 10^7$
Extreme ultra- violet	$3 \times 10^{15}$	$10^{-5} = 0.1\mu$	12.4	$2.1 \times 10^8$
Very soft X-rays...	$3 \times 10^{17}$	$10^{-7} = 10\text{\AA}$	1,240	$2.1 \times 10^9$
X-rays ... ..	$3 \times 10^{18}$	$10^{-8} = 1\text{\AA}$	12,400	$(m = 1.001 \times m_0)$ approaching speed of light
Hard X-rays ... ..	$3 \times 10^{19}$	$10^{-9} = 0.1\text{\AA}$	124,000	
$\gamma$ -rays ... ..	order $10^{21}$	order $10^{-3}\text{\AA}$	order $10^6$	

## HIGH-FREQUENCY EFFECTS.

If, however, we consider rays of high frequency, such as X-rays, which are now known to be periodic radiations far on the ultra-violet side of the spectrum, with wavelengths thousands of times shorter than those of the visible rays, we find that these three kinds of secondary radiations, namely, the diffused, fluorescent and corpuscular rays, present properties of a much more simple and fundamental kind.

Fluorescent rays, in particular, become quite characteristic of the atomic nature of the chemical elements present. When the incident rays are of a sufficiently high frequency, the emitted radiation shows at once the chemical nature of the illuminated substance and effects an immediate spectrum analysis of bodies upon which it falls. Each kind of atom shows its own spectral lines, the type of which is the same for all known elements, from hydrogen to uranium, the passage from one element to the next in the Mendeléeff table being followed by a regular shift of

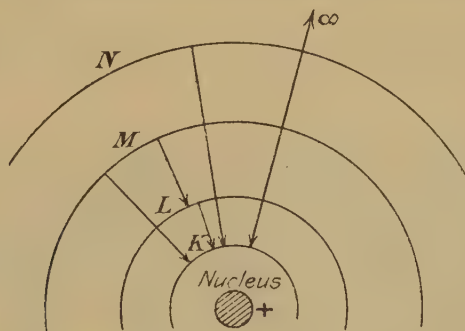


FIG. 2.—SIMPLE SCHEME OF LEVELS IN THE RUTHERFORD-BOHR ATOM.

the characteristic spectrum, according to the general and admirable law discovered by Moseley in 1913.

Photo-electric rays, in this part of the spectrum, play an important role ; they possess an energy corresponding to many thousands of volts and their effects are readily perceived. Arriving from the internal parts of the atom, these corpuscles give us news of the mysterious regions from which they come and offer a fascinating field of study.

## THE ATOMIC MODEL.

We must now glance at the present conceptions of atomic structure, in order to see how they can offer an interpretation, or at least a rough representation, of the different effects of which we are speaking.

As a consequence mainly of the remarkable work of Sir Ernest Rutherford and Professor Bohr, atoms are now generally considered as consisting of a central positive part, the nucleus, surrounded by a certain number of electrons rotating around it and belonging to a certain number of shells, or levels, so defined that a definite amount of work is required to expel from the atom an electron belonging to a given level. This work of extraction increases the deeper the electron is within the atom.

These levels are called the  $K, L, M \dots$  levels, and the corresponding energies of extraction denoted by  $W_K, W_L, W_M \dots$ , these values being directly connected with the frequency of the absorption bands or limits, now well known to be general and regular features of X-ray spectra. For instance, we have for the  $K$  limit the frequency  $\nu_K = \frac{W_K}{h}$ ,  $h$  being Planck's constant.

The action of the light is considered to take place as follows: When radiation with a frequency  $\nu$  strikes the atom, it penetrates, so to speak, as far as the lowest level if the work of extraction  $W_K$  of its electrons is inferior to the quantum of the radiation. If the frequency lies between the  $K$  and  $L$  absorption limits of the atom ( $\nu_L < \nu < \nu_K$ ), the radiation will be capable of reaching the  $M$  and  $L$  electrons, but not the  $K$  ones. Every excited electron starts off with an initial energy equal to the quantum  $h\nu$  of the exciting ray and retains, when out of the atom, a residual speed corresponding to the energy  $h\nu - W$ .

Bohr's theory regards the fluorescent rays as a consequence of the passage of an electron from a certain level to an inner one when there is a free place on that inner level. Radiation is then emitted with a frequency equal to

$$\frac{W_f - W_i}{h}$$

It is seen that this view involves the condition, in order that fluorescent rays may be produced, that there should be an empty place on a certain level; and we know that such free places are the result of photo-electric action due to an exciting external (or internal) radiation. The detailed study of photo-electrons allows us to gain information in regard to the Bohr levels, and, consequently, to discover a series of data which are the key to all possible emission phenomena relating to the atom under consideration.

#### EXPERIMENTAL METHODS.

Let us suppose an apparatus including a highly exhausted vessel in which is contained a secondary radiator formed of silver foil, for instance, and that we illuminate the foil with a beam of X-rays. The silver atoms will lose some of their electrons, which will start off with different speeds and, provided there is a suitable magnetic field, will be deflected so as to form a series of lines on a photographic plate.

The results may be summarised as follows: A given monochromatic incident beam of X-rays of frequency  $\nu_0$  gives rise to a stream of electrons of a definite velocity equal to  $h\nu_0 - W$ , from each Bohr level of the atom of the secondary radiator; the "velocity spectrum" when measured yields the work of extraction of every level in the excited atom.

On the other hand, if we consider a given level of the secondary radiator on which a complex beam of X-rays falls, each X-ray component will give a definite line, so that the velocity-spectrum now becomes a real X-ray spectrum, obtained (without any use of a crystal) by means of a magnetic field.

Similar results can be obtained with the gamma-rays of radio-active substances.

We have thus surveyed an experimental device suited to the quantitative study of photo-electrons. But I would like to mention another which allows us to

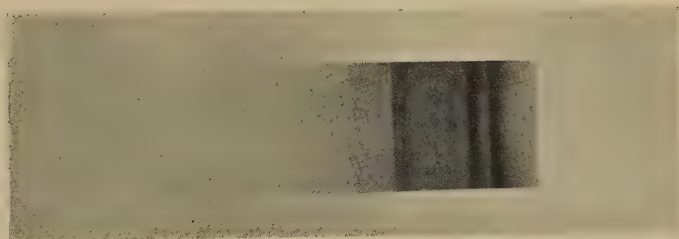


FIG. 3.—CORPUSCULAR SPECTRUM OF SILVER, USING K-RAYS OF TUNGSTEN (DE BROGLIE).

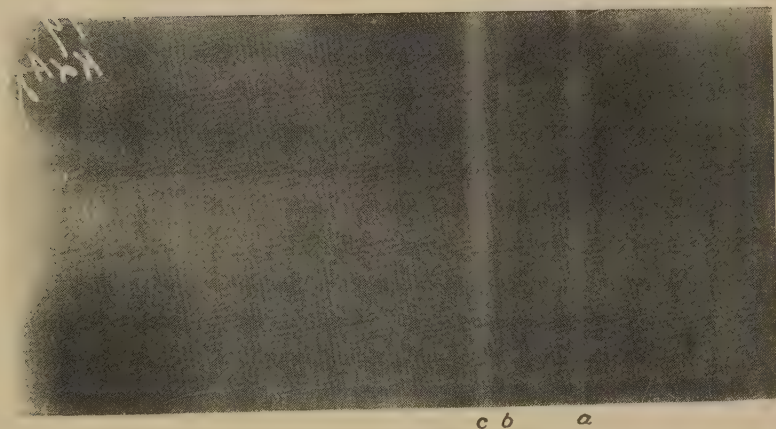


FIG. 5.—CHANGE OF WAVE-LENGTH BY DIFFUSION. K-RAYS OF SILVER FALLING ON A GRAPHITE RADIATOR (DE BROGLIE AND A. DAUVILLIER).

- (a) K-doublet serving to standardise the plate.
- (b) K-doublet of silver diffused without change of wave-length (the lines remain sharp).
- (c) K-doublet displaced (the Compton effect).

*To face page 426.]*





observe directly, so to speak, the expulsion of photo-electrons by the Röntgen rays. This is the very remarkable method of Professor C. T. R. Wilson, a description of which would, I am sure, be quite superfluous. Everybody knows that if a beam of rays is admitted into an expansion chamber where a moist gas may be cooled by a sudden increase of volume, a slight fog is manifested, and that the fog droplets are mainly produced where the atoms of the gas are electrified by their decomposition into ions; that is to say, along the tracks of the photo-electrons, which thus become visible, forming sharp curved lines easily registered on a photographic plate.

It is always highly interesting to consider such plates. At least two general classes of tracks are clearly evident—the long ones, corresponding to photo-electrons connected with the emission of fluorescent rays, and some quite short ones which cannot be explained in such a way.

#### HIGH-FREQUENCY DIFFUSION.

Before concluding my lecture, I should like to say a word on that type of secondary rays, which received no more than a passing mention at the outset, namely, the diffused rays.

Recent experiments in the United States, due to Professor A. H. Compton,

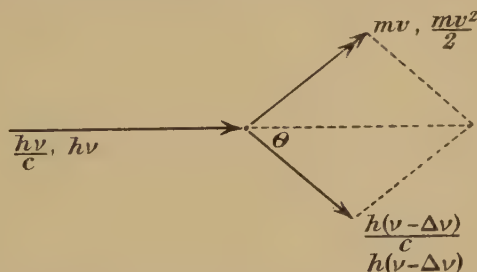


FIG. 4.—DIAGRAM ILLUSTRATING THE CHANGE IN WAVE-LENGTH AT DIFFUSION.

have shown that, at least in the case of X-rays, the diffused secondary rays are not, as previously believed, identical in nature with the exciting rays. They possess a longer wave-length, the difference of wave-length between the primary and secondary rays varying with what may be called the diffusion angle.

For instance, Compton has shown that with an incident beam consisting of the molybdenum *K* lines, having a wave-length of about 0.7 Ångstrom units, the shift may be of the order of 3 per cent.

Debye and Compton, in order to account for these results, have proposed a theory based on quantum considerations. Considering the incident ray, the diffused ray, and the electron which is supposed to participate in the action, these three elements are regarded as amenable to the principles of the conservation of energy and of momentum. The result of the calculation is expressed by the formula

$$\Delta\lambda = \frac{2h}{mc} \sin^2 \frac{\theta}{2} = 0.0484 \sin^2 \frac{\theta}{2}$$

the change of wave-length being in Ångstrom units, and  $\theta$  denoting the angle between the incident and diffused rays.

These experimental results are quite novel, and certainly somewhat extraordinary; the theoretical explanation proposed, although very tempting, no doubt needs cautious examination. Quite recently some doubt has been thrown by other experimenters on the validity of Compton's results, and the question may in a certain sense be considered as still an open one.\* I would like to say that I have tried some experiments on the point, and have worked especially with Tungsten lines ( $\lambda=0.2 \text{ \AA}$ ) incident on a secondary radiator of a special form. M. Dauvillier working in my laboratory has also used an improved apparatus with a silver target illuminating various types of radiators. The result (Fig. 5) is a confirmation of Compton's views, and I believe we have in this phenomenon a new and very suggestive field of research.

#### CONCLUSION.

To summarise, I have attempted to show in this lecture how the action of high-frequency radiation on matter, with production of the three kinds of secondary rays, affords a valuable experimental field for the study of both matter and light, and I shall have succeeded in my main object if I have made this special point clear. Let me add how much I appreciate this opportunity of speaking on a subject to which the contributions of British physicists have been both brilliant and important.

\* See Clark and Duane, Proc. Nat. Acad. of Science, Washington, Nov. (1923), and Jan. and Feb. (1924).

DEMONSTRATION OF A METHOD OF OPTICAL PROJECTION OF  
OPAQUE OBJECTS.

By W. A. BENTON.

THE object of the apparatus demonstrated is to provide means for opaque projection of a sufficiently simple design to be within the means of schools. The lens is calculated to have as wide an angle as possible, other merits being sacrificed to this one so far as is necessary. In the result very strong illumination is obtained, and chromatic and spherical aberration are not noticeable in the conditions for which the device is intended.

The object to be projected is illuminated by two gas-filled incandescent lamps, and the light from it falls on a specially designed simple condensing lens of wide angle, whence it passes to the screen through a lens of smaller aperture which is chromatically overcorrected.



## DEMONSTRATION OF AN ANOMALY IN FRICTIONAL ELECTRICITY.

By Prof. A. O. RANKINE, *O.B.E., D.Sc.*, Imperial College of Science and Technology.

THE effects shown were first noticed by Mr. T. Banfield during a lecture experiment. They concern the electrification of two ebonite rods made from the same piece, the one polished and the other unpolished, by friction on pieces of Government flannel. For rapidly discharging these objects for purposes of successive tests the ionisation produced by an X-ray tube was employed, and the signs of the charges were tested by means of an electroscope to which a positive charge had been transferred from an electrophorus.

(a) With the flannel as purchased the usual effect of negative electrification is produced on both rods, whether the friction is effected by rubbing the rod with the flannel held in the hand or by beating the suspended flannel with the rod. This flannel is, however, an imperfect insulator. When it has been thoroughly washed and dried it becomes highly insulating, and the following effects are produced : (b) If the flannel is suspended and beaten vigorously a number of times with the polished rod the latter at first acquires the usual negative charge. (c) When the beating is continued, however, the rod acquires eventually a large *positive* charge. (This disconcerting effect occurred in the first instance after the students had been confidently assured that the reverse would be the case, the ebonite rod violently repelling a suspended glass rod electrified by friction with silk.) The phenomenon occurs even when, between each stroke, flannel and rod are completely discharged by ionisation. The polished ebonite appears to pass gradually under the treatment from a negative to a positive condition with respect to the flannel. (d) When once the rod has reached this condition and has been discharged, beating is no longer necessary ; it will again acquire a positive charge when *rubbed hard* with other prepared pieces of flannel, and not only with the piece which has been beaten ; but the flannel must have been prepared by washing and drying. This seems to suggest that a change in the rod, not in the flannel, is produced by the prolonged and vigorous friction involved in the beating. (e) When the rod is rubbed *lightly* with the prepared flannel after the above treatment it becomes negatively charged, but subsequent vigorous rubbing reproduces the positive electrification. At this stage either negative or positive electrification can be obtained at will, according to the vigour of the friction. This condition of the ebonite, however, is not permanent. Rubbing with unprepared poorly insulating flannel, for example, will quickly restore the rod to the original state, starting again from which the whole sequence of events may be repeated. Two different polished rods exhibit the same behaviour. (f) The unpolished rod does not give these effects ; it always becomes negatively charged whatever the mode of friction with the prepared flannel. (g) Yet it is remarkable that when the polished and unpolished rods are rubbed together it is the *unpolished* rod that becomes *positive* and the *polished* rod *negative*.

No explanation is yet offered, but it is clear that in frictional electrification attention has to be paid to the mode of execution ; the constitution of the substances is not the sole determining factor in the distribution of the charges.

DEMONSTRATION OF THE PHOSPHORESCENCE OF FUSED  
TRANSPARENT SILICA.

By Dr. W. E. CURTIS, King's College, London.

THE phenomenon, to which attention has been called by various observers, was demonstrated with a mercury lamp and a hydrogen tube, both of transparent silica. The effect of heat in increasing the intensity of the phosphorescence and decreasing its duration was also shown.

Observations made on a silica tube containing helium and a very slight trace of hydrogen were described. It was found that whereas a condensed (oscillatory) discharge gave the Balmer lines faintly and feeble phosphorescence, an uncondensed (unidirectional) discharge developed neither Balmer lines nor phosphorescence. There was also a correspondence between the localisation of the Balmer lines and that of the phosphorescence. On admitting hydrogen to the helium tube by heating it red hot in a blowpipe flame the phosphorescence was obtained strongly. These observations suggested that the effect might be due to radiation of wavelengths occurring in the hydrogen spectrum, but absent from the helium spectrum, and it was pointed out that the Lyman series of hydrogen, which lies between  $\lambda\lambda$  1,215-912Å, was probably the effective radiation, since there are in this region no helium lines. Some support is lent to this view by the fact of the strong absorption of quartz in the region in question, the centre of its absorption band being situated at about  $\lambda$  1,050Å. The other gases whose radiations are effective in producing phosphorescence are also believed to give rise to lines in this region.

## DEMONSTRATION OF A SELENIUM PHOTOMETER.

By Dr. F. C. Toy.

THIS instrument was designed for use by photographic manufacturers in measuring the density of a negative after test exposures which vary from strip to strip of the negative. The intensities of two beams of light are compared, one of which passes through the part of the negative to be tested while the other passes through a compensating wedge which has to be adjusted until its effect on the second beam is equal to that of the negative on the first beam. A single lamp is employed, the light from which falls on two circular opals, each about 5 mm. in diameter, which are arranged on opposite sides of the lamp and serve as the sources of the two beams. The latter are deflected by means of prisms, one passing through the negative while the other passes through the compensating wedge; both eventually fall on a selenium cell in the circuit of which is a sensitive galvanometer. Two shutters, one for each beam, are mounted at opposite ends of a pivoted arm and are so designed that as one beam is progressively obscured by one shutter, the other beam is progressively admitted by the other, the total illumination of the selenium cell remaining constant provided that the two beams on reaching it are of equal intensity. The wedge is therefore adjusted until, with the required negative in position, there is no change in the deflection of the galvanometer on turning the shutter. The method is extremely accurate. If the opacity of the negative be 10, cutting the light-intensity down to 1/10th of its value so that the density, or logarithm of the opacity, is 1, extreme readings differ by 0.002. For a density of 5 the difference is less than 0.01.

## DISCUSSION.

Prof. A. O. RANKINE inquired whether errors might not arise through inequality of illumination in the two opals, which were located on opposite sides of the lamp.

Mr. R. S. WHIPPLE suggested that if that were so the errors might be allowed for by turning the lamp through 180 degrees, a method used by Dr. Moll to overcome a similar difficulty.

Dr. TOY, in reply to the discussion, said that the opal on the wedge side was purposely ground thinner than the other opal in order that it might be more strongly illuminated. Before taking observations it was necessary to adjust the scale until, with the wedge in zero position and with the negative removed, both beams were of equal intensity. Thus for a zero reading a certain thickness of wedge would be interposed and would compensate for any difference in brightness between the two opals. This seemed to be by far the simplest way of making the necessary allowance.

# DEMONSTRATION OF A NEW FORM OF STRING GALVANOMETER.

By Prof. ERNEST WILSON.

THE construction of the present instrument arose out of a consideration of the magnetic circuit of an instrument belonging to Prof. R. S. McDowell, in which two cobalt chromium steel magnets are united at each end by massive wrought-iron pole-pieces. It was found that the magnetic force in the gap was 1,340 C.G.S. units with both magnets in position, and 893 units when one magnet was removed.

A careful examination of the magnetic circuit showed that the magnets were seriously demagnetised. In the present instrument an attempt has been made to combine constancy and strength of the magnetic field with lightness. The magnets consist of two rings of cobalt chrome steel of 25 cm. diameter, the cross-section of the bar being 5 cm.  $\times$  1 cm.

Each ring has an air gap of 0.1 cm., and the poles have a bevil of 45 deg., terminating in opposing faces of 0.4 cm. wide. The magnets are mounted one above the other, with their gaps in line, and a separation between the rings of 1 cm.

The strings, two in number, are of platinum wire 0.002 cm. diameter, and are supported symmetrically in the gap by a brass former, the effective length of each string being 10 cm.

A beam of light is directed tangentially to the rings by a condensing lens, which focuses it on the strings; it is then reflected by a small prism, placed between the rings, and passes out at right angles through an objective which, with an eye-piece, gives a magnification of about 45 diameters. The image of the strings can be observed on a screen, or photographed.

As received the rings were in the hardened state but unmagnetised, and the tips of one of them were annealed before magnetisation.

There was sufficient spring in the steel magnets to allow of the insertion of distance pieces of known thickness; thus it was possible to investigate the effect of varying the length of the gap. The following figures were obtained with a search-coil :—

Thickness of distance piece in cms.	...	...	0.11	0.16	0.24
Magnetic force in gap	...	...	7230	6250	4910

The variation of magnetic force with smaller gaps was investigated by the magnetic potentiometer, and its rate of growth was found to increase rapidly with diminution of gap, until finally it had the value of 22,800, with a gap length of 0.012 cm. From the known properties of the steel it was inferred that when the pole faces were in contact the force was of the order of 23,000 C.G.S.

The leakage coefficient, which had an initial value of about unity, increased rapidly as the gap was increased from zero, and its rate of growth was still considerable in the later stages.



The author wishes to acknowledge the kindness of Mr. W. B. Thorpe, engineer-in-chief of the Charing Cross Electric Supply Co., in allowing the use of one of his immense storage-cell batteries, and the help of Mr. Mason in magnetising the rings.

#### DISCUSSION.

Mr. R. S. WHIPPLE congratulated Prof. Wilson on the simplicity of his design, and inquired as to the sensitivity of the instrument. He mentioned the case of an instrument which he had examined, in which two ring magnets of the d'Arsonval type had been spoilt by the defectiveness of a third similar magnet with which they were associated, serious demagnetisation being the result.

Dr. G. B. BRYAN asked whether the instrument had a temperature-coefficient arising from variations in the width of the air gap with temperature.

In reply to Dr. Bryan, Prof. WILSON said that the width of the gaps in the two rings was rigidly maintained constant by a massive brass plate which was bolted to the polar extremities. No magnetic material was used in the construction of the instruments beyond the rings themselves, and they were separated by a distance of 1 cm.

DEMONSTRATION OF A TWO-DIMENSIONAL RECORDING ACCELEROMETER FOR AEROPLANE RESEARCH.

By Dr. G. F. C. SEARLE, F.R.S.

THE instrument contains two accelerometer units, *V* and *H*. In each unit the moving element consists of a staff carrying a silver vane and a plane mirror and strung in a cradle by two torsion wires. The mass of each element is about 2 grammes. Each staff is perpendicular to its vane. The two staffs are parallel to each other and to the base of the instrument. The undamped periodic time is  $1/35$  sec.

When the base of the instrument is placed on a horizontal table, the centre of mass of the *V* element is nearly in the same horizontal plane as its staff, and the centre of mass of the *H* element is vertically below its staff.

Each vane moves between the poles of an electromagnet; the motion is approximately dead-beat.

The movements of the vanes are recorded photographically. In the case of each element, rays from an illuminated slit, after passing through a lens, are reflected by the mirror carried by the staff on to a kinematograph film which is drawn past a slit *S* by clockwork. The centres of the mirrors lie in the plane containing *S* and perpendicular to the staffs, and thus the movement of each vane is recorded on the same film. The slit *S* is vertical when the base of the instrument is horizontal.

On moving a lever by hand, the clockwork is started, the lamps for illuminating the slits are caused to glow and the damping electromagnets are energised. The same effect can be produced electromagnetically by closing a switch at any distance from the instrument. A third lamp, which can be caused to flash by depressing a key, is used to make a mark across the film at any desired instant.

The zero of each element is recorded photographically by placing the instrument so that the corresponding staff is vertical. If the box be then placed in turn in each of the two positions, in which the staff is horizontal and the line joining the centre of mass of the element to the axis of the staff is nominally horizontal, two other marks will be recorded. The distance of each mark from the zero is *a* cm. for element *V* and *b* cm. for *H*; approximately,  $a=b=0.4$  cm. In the case of *V*, the torsion wires exert on the staff a force and a couple which together are equivalent to a force  $mg$  dynes acting at the centre of mass of the element of mass *m* grammes, perpendicular to the base of the instrument.

If at any instant a point on the *V* record is at a distance *na* from the zero line for *V*, the action of the torsion wires on the *V* staff is equivalent to a force, acting at the centre of mass of the element, having a component  $mng$  dynes perpendicular to the base of the instrument, whether the base be horizontal or not. Since the motion of the element is dead beat, its centre of mass has the same acceleration in this direction as the aeroplane which carries the instrument, and

it follows that each gramme of the aeroplane is acted on by a non-gravitational force of  $ng$  dynes in the specified direction. Hence the force exerted by the air upon the aeroplane in that direction is  $nMg$  dynes, where  $M$  grms. is the mass of the aeroplane. Since the wings are of relatively small mass and the air exerts a comparatively small force on the body of the aeroplane,  $nMg$  dynes is nearly the same as the force exerted by the wings upon the body in the specified direction. Records published by H.M. Stationery Office show that  $n$  may be as great as 4.

The record given by the unit  $H$  is interpreted in a similar manner. This unit will show, for instance, the horizontal force exerted by the water upon a sea-plane when it alights upon the sea.

Records obtained by a single accelerometer element fitted into a recording instrument lent by the Royal Aircraft Establishment were shown. They exhibited the calibration of the instrument, the effect of dropping it, and also the effects produced when the observer walked or jumped with the instrument. [August 1, 1924. Since the Meeting on July 19, the two-dimensional instrument has been adjusted and twin records have been obtained.]

The two-dimensional instrument has been constructed by the Cambridge Instrument Co., Ltd., for the Belgian Government.

It is hoped that a full description of the instrument will be published in the "Journal of Scientific Instruments."

# INDEX.

## A

	PAGE
Absorption and Scattering of $\gamma$ -Rays, The .....	355
Accelerometer for Aeroplane Research, A Two-Dimensional Recording (Demonstration) .....	435
Aolian Tones .....	153
Aerodynamic Resistance of Spheres, Shot Upwards to Measure the Wind, The .....	67
Andrews, J. A. ( <i>see</i> Lees, Prof. C. H.)	
Atomic Structure ( <i>see</i> Structure, Atomic)	
Awbery, J. H. ( <i>with</i> Griffiths, Dr. E.), <i>on</i> Apparatus for the Determination of the Heat of Evaporation of Liquids of High Boiling Points.....	303

## B

Bairsto, Dr. G. E., <i>On</i> a Method for the Synchronous and Instantaneous Illumination of Objects Rotating or Vibrating at Very High Speeds.....	349
Benton, W. A., <i>Demonstration by</i> .....	429
Bismuth Alloys, <i>On</i> the Thermo-electric Properties of .....	281
Bond, W. N., <i>Demonstration by</i> .....	340
Bond, W. N., <i>on</i> The Flow of Compressible Fluids, Treated Dimensionally .....	367
Brain, K. R., <i>on</i> Investigations of Piezo-electric Effects with Dielectrics .....	81
Brogie, M. le Duc de ( <i>see</i> Guthrie Lecture, Ninth)	

## C

Calthrop, J. E., <i>on</i> The Effects of Torsion on the Thermal and Electric Conductivities of Metals .....	168
Capacities, A Method of Measuring Very Small .....	399
Chadwick, Dr. J. ( <i>see</i> Rutherford, Sir Ernest)	
Chatley, Dr. H., <i>on</i> Cohesion .....	336
Clack, Dr. B. W., <i>On</i> the Study of Diffusion in Liquids by an Optical Method .....	313
Clarkson, W. ( <i>see</i> Taylor, J.)	
Cohesion .....	336
Conductivities of Metals, Effects of Torsion on the Thermal and Electrical.....	168
Critical Resistance for Flashing of the Low-voltage Neon Discharge Tube, A.....	269
Crystal Structure ( <i>see</i> Structure, Crystal)	
Curtis, Dr. W. E., <i>Demonstration by</i> .....	431

## D

Darling, C. R. ( <i>with</i> Rinaldi, R. H.), <i>On</i> the Thermo-electric Properties of Bismuth Alloys, with special reference to the Effect of Fusion. With a note on Thermo-electric Reinversion .....	281
Deodhar, D. B., <i>on</i> Israj, a Remarkable Indian Stringed Instrument .....	379



	PAGE
Demonstrations :—	
Anomaly in Frictional Electricity, An .....	430
Auxiliary Optical System for the Examination of Crystal Sections by Conoscopic Methods, An.....	210
Fluorescence of Transparent Fused Silica, The .....	431
New String Galvanometer, A .....	433
Optical Projection of Opaque Objects, A Method of .....	429
Reproduction of Vowel Sounds, Experiments on the .....	45
Selenium Photometer, A .....	432
Sub-harmonics Produced by a Tuning-fork .....	340
Two-Dimensional Recording Accelerometer for Aeroplane Research, A .....	435
“ Diametral Plane ” in Elementary Optics, The .....	294
Dielectrics, Investigations of Piezo-electric Effects with .....	81
Diffusion in Liquids by an Optical Method, On the Study of.....	313
Discussion on Loud-Speakers for Wireless and Other Purposes .....	114, 211
Disintegration of Atomic Nucleus ( <i>see</i> Structure, Atomic)	

## E

Electrical Properties of the Neon Lamp, Notes on Some .....	262
Evans, J. C. ( <i>see</i> Shaxby, J. H.)	
Evaporation of Liquids of High Boiling Points, Apparatus for the Determination of the Heat of .....	303
Excitation and Ionisation Potentials of Gases and Vapours .....	1

## F

Fage, Miss W. E. ( <i>see</i> Owen, Dr. E. A.)	
Ferguson, Dr. Allan, On the Measurement of the Surface Tension of a Small Quantity of Liquid .....	37
Fleming, N. ( <i>see</i> Owen, Dr. E. A.)	
Flow of Compressible Fluids, Treated Dimensionally, The .....	367
Fluorescence of Transparent Fused Silica (Demonstration) .....	431
Frictional Electricity, An Anomaly in (Demonstration) .....	430

## G

Galvanometer, A New String (Demonstration) .....	433
$\gamma$ -Rays, The Absorption and Scattering of .....	355
Gas-grown Skin, A Preliminary Measurement of a Primary .....	288
Gas-grown Skins from a Sprengel Pump, The Removal of.....	291
Griffiths, Dr. Ezer ( <i>see</i> Awbery, J. H.)	
Grindley, E. N. ( <i>see</i> Piper, S. H.)	
Guthrie Lecture, the Ninth : M. le Duc de Broglie on The Photo-electric Effect in the case of High-frequency Radiation, and some associated Phenomena .....	423

# Index.

439

## H

	PAGE
Hartshorn, L., <i>on</i> A Method of Measuring Very Small Capacities .....	399
Hawksley, C. W., Demonstration by .....	210
Heat of Evaporation of Liquids of High Boiling Points, Apparatus for the Determination of the .....	303
Horton, Dr. F. : <i>Lecture on</i> The Excitation and Ionisation Potentials of Gases and Vapours .....	1

## I

Illumination of Objects Rotating or Vibrating at Very High Speeds, A Method for the Synchronous and Instantaneous.....	349
Intermetallic Compounds, The Atomic Structure of Two .....	341
Ionisation Effect, A New Photo-electric and .....	241
Ionisation Potentials of Gases and Vapours, The Excitation and.....	1
Israj, a Remarkable Indian Stringed Instrument, Note on .....	379

## L

Latent Heat ( <i>see</i> Heat of Evaporation)	
Lees, Prof. C. H., <i>on</i> The Diametral Plane in Elementary Optics .....	294
Lees, Prof. C. H. ( <i>with</i> Andrews, J. A., and Shave, L. S.), <i>on</i> The Variation of Young's Modulus at High Temperatures .....	405
Light, Scattering of, by $\text{CO}_2$ , $\text{N}_2\text{O}$ , and some Organic Vapours.....	32
Liquids, Diffusion in .....	313
Liquids, Heat of Evaporation of .....	303
Liquids, Surface Tension of .....	32, 205
Loud-Speakers for Wireless and Other Purposes, Discussion on .....	114, 211

## M

Magnetic Field, The Effect of, on the Surface Tension of a Liquid of High Susceptibility .....	205
Manley, J. J., <i>on</i> A Preliminary Measurement of a Primary Gas-grown Skin .....	288
Manley, J. J., <i>On</i> the Removal of Gas-grown Skins from a Sprengel Pump .....	291
Milne, E. A. : <i>Lecture on</i> Recent Work in Stellar Physics .....	94
Modulus at High Temperatures, The Variation of Young's.....	405

## N

Narayan, A. L., <i>on</i> the Scattering of Light by $\text{CO}_2$ , $\text{N}_2\text{O}$ , and some Organic Vapours .....	32
Neon Lamps, Properties of .....	241, 253, 262, 269

## O

Optical Method, <i>On</i> the Study of Diffusion in Liquids by an .....	313
Optical Projection, A Method of (Demonstration) .....	429
Optical System for Conoscopic Examination of Crystal Sections (Demonstration) .....	210

	PAGE
Optics, The "Diametral Plane" in Elementary .....	294
Oschwald, U. A. ( <i>with</i> Tarrant, A. G.), on A New Photo-electric and Ionisation Effect .....	241
Oschwald, U. A., Notes On some Electrical Properties of the Neon Lamp .....	262
"Osglim" Neon-filled Lamp, On Certain Properties of the.....	253
Owen, Dr. E. A. ( <i>with</i> Preston, G. D.), on The Atomic Structure of Two Inter-metallic Compounds .....	341
Owen, Dr. E. A., on X-Ray Analysis of Solid Solutions .....	14
Owen, Dr. E. A., on X-Ray Analysis of Zinc-Copper Alloys .....	49
Owen, Dr. E. A. ( <i>with</i> Fleming, N., and Fage, Miss W. E.), on The Absorption and Scattering of $\gamma$ -Rays.....	355

## P

Paget, Sir Richard, Demonstration by .....	45
Pettersson, Dr. Hans, On the Structure of the Atomic Nucleus and the Mechanism of its Disintegration .....	194
Photo-electric Effect in the case of High-Frequency Radiation, and some Associated Phenomena, The.....	423
Photo-electric and Ionisation Effect, A New .....	241
Photometer, A Selenium (Demonstration) .....	432
Piezo-electric Effects with Dielectrics, Investigations of.....	81
Piper, S. H. ( <i>with</i> Grindley, E. N.), on The Structure of Sodium Salts (Discussion)	31
Preston, G. D. ( <i>see</i> Owen, Dr. E. A.)	

## R

Radiation, High-Frequency :—	
Absorption and Scattering of $\gamma$ -Rays, The.....	355
Photo-electric Effect in the case of High-Frequency Radiation, and some Associated Phenomena, The (Guthrie Lecture) .....	423
Rankine, Prof. A. O., Demonstration by.....	430
Relativity, A Generalisation of Prof. Whitehead's Theory of.....	176
Richardson, E. G., on Aeolian Tones.....	153
Richardson, L. F., on The Aerodynamic Resistance of Spheres shot upward to Measure the Wind .....	67
Richardson, Prof. O. W., on Thermionic Emission from Systems with Multiple Thresholds .....	383
Rinaldi, R. H. ( <i>see</i> Darling, C. R.)	
Rolton, Miss Winifred L. ( <i>with</i> Troop, R. S.), on The Effect of a Magnetic Field on the Surface Tension of a Liquid of High Susceptibility .....	205
Rutherford, Sir Ernest ( <i>with</i> Chadwick, Dr. J.), on Recent Experiments on the Artificial Disintegration of the Elements .....	417

## S

Scattering of $\gamma$ -Rays, The Absorption and .....	355
Scattering of Light by $\text{CO}_2$ , $\text{N}_2\text{O}$ , and some Organic Vapours.....	32
Searle, Dr. G. F. C., Demonstration by .....	435
Selenium Photometer (Demonstration).....	432

	PAGE
Shave, L. S. ( <i>see</i> Lees, Prof. C. H.)	
Shaxby, J. H. ( <i>with</i> Evans, J. C.), On Certain Properties of the "Osglim" Neon-filled Lamp .....	253
Silica, The Fluorescence of Transparent Fused (Demonstration) .....	431
Skins, Gas-grown .....	288, 291
Sodium Salts, The Structure of (Discussion) .....	31
Solid Solutions, X-Ray Analysis of .....	14
Spheres Shot Upward to Measure the Wind, The Aerodynamic Resistance of .....	67
Stellar Physics, Recent Work in (Lecture) .....	94
Sub-harmonics produced by a Tuning Fork (Demonstration) .....	340
Stringed Instrument, Note on Israj, A Remarkable Indian .....	379
Structure, Atomic :—	
Recent Experiments on the Artificial Disintegration of the Elements .....	417
Structure of the Atomic Nucleus and the Mechanism of its Disintegration, On the .....	194
Structure, Crystal :—	
Intermetallic Compounds, The Atomic Structure of Two .....	341
Sodium Salts, The Structure of (Discussion) .....	31
Solid Solutions, X-Ray Analysis of .....	14
Zinc-Copper Alloys, X-Ray Analysis of .....	49
Surface-tension .....	37, 205

## T

Tarrant, A. G. ( <i>see</i> Oschwald, U. A.)	
Taylor, J. ( <i>with</i> Clarkson, W.), on A Critical Resistance for Flashing of the Low-voltage Neon Discharge Tube .....	269
Temple, G., on A Generalisation of Professor Whitehead's Theory of Relativity...	176
Thermionic Emission from Systems with Multiple Thresholds .....	383
Thermo-electric Properties of Bismuth Alloys, with special reference to the Effect of Fusion, On the .....	281
Thermo-electric Reversion, A Note on .....	281
Thresholds, Thermionic Emission from Systems with Multiple .....	383
Tones, Aeolian .....	153
Torsion, The Effects of, on the Thermal and Electric Conductivities of Metals .....	168
Toy, Dr. F. C., Demonstration by .....	432
Troop, R. S. ( <i>see</i> Rolton, Winifred L.)	

## V

Vowel Sounds, Reproduction of (Demonstration) .....	45
---	----

## W

Whitehead's Theory of Relativity, A Generalisation of Prof. ....	176
Wilson, Prof. Ernest, Demonstration by .....	433
Wireless and Other Purposes, Discussion on Loud-Speakers for .....	114, 211



## X

X-Ray Analysis (*see* Structure, Crystal)

PAGE

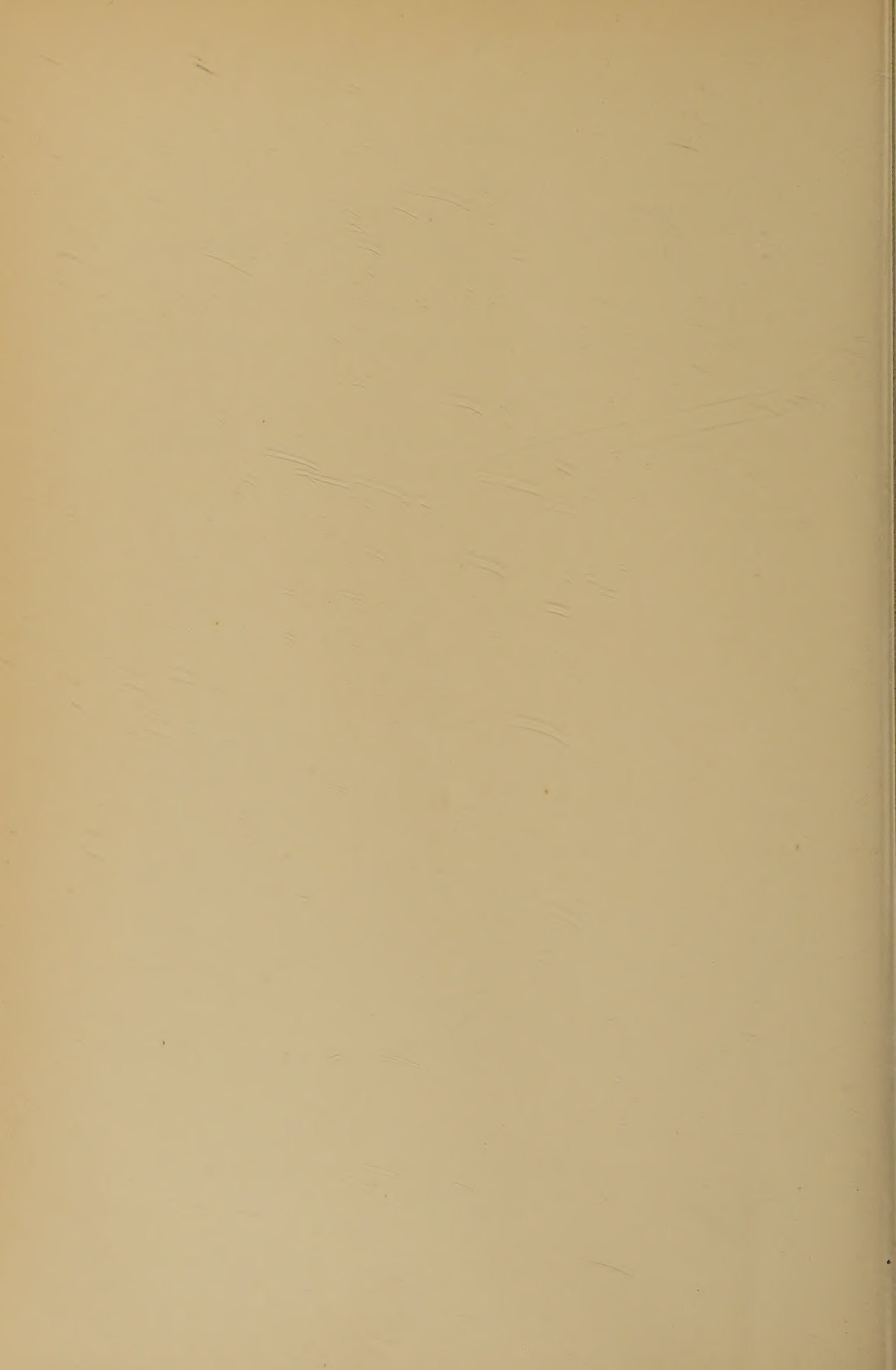
## Y

Young's Modulus at High Temperatures, The Variation of ..... 405

## Z

Zinc-Copper Alloys, X-Ray Analysis of ..... 49







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## CONTENTS.

	PAGE
Title, Contents, Proceedings at Meetings .....	i.-xxii.
XXIII. The Atomic Structure of Two Intermetallic Compounds. By E. A. OWEN, M.A., D.Sc., and G. D. PRESTON, B.A. ....	341
XXIV. On a Method for the Synchronous and Instantaneous Illumina- tion of Objects Rotating or Vibrating at Very High Speeds. By G. E. BAIRSTO, D.Sc., D.Eng., F.Inst.P. ....	349
XXV. Absorption and Scattering of Gamma-Rays. By E. A. OWEN, M.A., D.Sc., N. FLEMING, B.A., and WINIFRED E. FAGE, B.Sc. ....	355
XXVI. The Flow of Compressible Fluids, Treated Dimensionally. By W. N. BOND, B.A. (Cantab.), M.Sc. (Lond.), F.Inst.P. ....	367
XXVII. Note on Israj, a Remarkable Indian Stringed Instrument. By D. B. DEODHAR, M.Sc. ....	379
XXVIII. Thermionic Emission from Systems with Multiple Thresholds. By Prof. O. W. RICHARDSON, M.A., D.Sc., F.R.S. ....	383
XXIX. A Method of Measuring Very Small Capacities. By L. HARTS- HORN, A.R.C.S., B.Sc., D.I.C. ....	399
XXX. The Variation of Young's Modulus at High Temperatures. By CHAS. H. LEES, D.Sc., F.R.S., JAS. P. ANDREWS, B.Sc., and L. S. SHAVE, B.Sc. ....	405
XXXI. Further Experiments on the Artificial Disintegration of Elements. By Sir E. RUTHERFORD, F.R.S., and Dr. J. CHADWICK .....	417
The Ninth Guthrie Lecture: The Phenomena of High-Frequency Radiation. By M. LE DUC DE BROGLIE. ....	423
Demonstration of a Method of Optical Projection of Opaque Objects. By W. A. BENTON. ....	429
Demonstration of an Anomaly in Frictional Electricity. By Prof. A. O. RANKINE, O.B.E., D.Sc. ....	430
Demonstration of the Phosphorescence of Fused Transparent Silica. By Dr. W. E. CURTIS .....	431
Demonstration of a Selenium Photometer. By Dr. F. C. TOY .....	432
Demonstration of a New Form of String Galvanometer. By Prof. ERNEST WILSON .....	433
Demonstration of a Two-Dimensional Recording Accelerometer for Aeroplane Research. By Dr. G. F. C. SEARLE, F.R.S. ....	435
Index .....	437